the final results are not very sensitive."[3] The best models are simple, yet still manage to communicate powerful insights into how the real world operates. In this spirit, the models presented here make assumptions that clearly are not true but allow us to simplify the framework and make it easier to grasp key concepts and insights. For example, we begin by assuming that our prototype economy has one type of homogeneous worker and one type of capital good that combine to produce one standard product. No economy in the world has characteristics even closely resembling these assumptions, but making these assumptions allows us to cut through many details and get to the core concepts of the theory of economic growth.

THE BASIC GROWTH MODEL

The most fundamental models of economic output and economic growth are based on a small number of equations that relate saving, investment, and population growth to the size of the workforce and capital stock and, in turn, to aggregate production of a single good. These models initially focus on the *levels* of investment, labor, productivity, and output. It then becomes straightforward to examine the *changes* in these variables. Our ultimate focus is to explore the key determinants of the *change in output*—that is, on the rate of economic growth. The version of the basic model that we examine here has five equations: an aggregate production function, an equation determining the level of saving, the saving-investment identity, a statement relating new investment to changes in the capital stock, and an expression for the growth rate of the labor force.[4] We examine each of these in turn.

Standard growth models have at their core a production function. At the individual firm or microeconomic level, a production function relates the number of employees and machines to the size of the firm's output. For example, the production function for a textile factory would reveal how much more output the factory could produce if it hired (say) 50 additional workers and purchased five more looms. Production functions often are derived from engineering specifications that relate given amounts of physical input to the amount of physical output that can be produced with that input. At the national or economywide level, production functions describe the relationship of the size of a country's total labor force and the value of its capital stock with the level of that country's gross domestic product (GDP; its total output). These economywide relationships are called **aggregate production functions**.

[3]Robert Solow, "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics* 70 (February 1956), 65–94.

[4]This five-equation presentation is based on teaching notes compiled by World Bank economist Shantayanan Devarajan, to whom we are indebted.

Our first equation is an aggregate production function. If Y represents total output (and therefore total income), K is the capital stock, and L is the labor supply, at the most general level, the aggregate production function can be expressed as

$$Y = F(K, L) \qquad [4\text{–}1]$$

This expression indicates that output is a function (denoted by F) of the capital stock and the labor supply. As the capital stock and labor supply grow, output expands. Economic growth occurs by increasing either the capital stock (through new investment in factories, machinery, equipment, roads, and other infrastructure), the size of the labor force, or both. The exact form of the function F (stating precisely *how much* output expands in response to changes in K and L) is what distinguishes many different models of growth, as we will see later in the chapter. The other four equations of the model describe how these increases in K and L come about.

Equations 4–2 through 4–4 are closely linked and together describe how the capital stock (K) changes over time. These three equations first calculate total saving, then relate saving to new investment, and finally describe how new investment changes the size of the capital stock. To calculate saving, we take the most straightforward approach and assume that saving is a fixed share of income:

$$S = sY \qquad [4\text{–}2]$$

In this equation, S represents the total value of saving, and s represents the average saving rate. For example, if the average saving rate is 20 percent and total income is $10 billion, then the value of saving in any year is $2 billion. We assume that the saving rate s is a constant, which for most countries is between 10 and 40 percent (typically averaging between 20 and 25 percent), although for some countries it can be higher or lower. China's savings rate in 2008 (along with those of several large oil exporters) exceeded 50 percent, while several countries (including Mozambique, Guinea, the Seychelles, and Georgia) reported savings rates less than 5 percent of GDP. Actual saving behavior is more complex than this simple model suggests (as we discuss in Chapter 10), but this formulation is sufficient for us to explore the basic relationships between saving, investment, and growth.

The next equation relates total saving (S) to investment (I). In our model, with only one good, there is no international trade (because everyone makes the same product, there is no reason to trade). In a closed economy (one without trade or foreign borrowing), saving must be equal to investment. All output of goods and services produced by the economy must be used for either current consumption or investment, while all income earned by households must be either consumed or saved. Because output is equal to income, it follows that saving must equal investment. This relationship is expressed as

$$S = I \qquad [4\text{–}3]$$

We are now in a position to show how the capital stock changes over time. Two main forces determine changes in the capital stock: new investment (which adds to the capital stock) and depreciation (which slowly erodes the value of the existing capital stock over time). Using the Greek letter delta (Δ) to represent the change in the value of a variable, we express the change in the *capital* stock as ΔK, which is determined as follows:

$$\Delta K = I - (dK) \qquad [4\text{–}4]$$

In this expression d is the rate of depreciation. The first term (I) indicates that the capital stock *increases* each year by the amount of new investment. The second term $-(d \times K)$ shows that the capital stock *decreases* every year because of the depreciation of existing capital. We assume here that the depreciation rate is a constant, usually in the range of 2 to 10 percent.

To see how this works, let us continue our earlier example, in which total income is $10 billion and saving (and therefore investment) is $2 billion. Say that the value of the existing capital stock is $30 billion and the annual rate of depreciation is 3 percent. In this example, the capital stock increases by $2 billion because of new investment but also decreases by $0.9 billion (3 percent × $30 billion) because of depreciation. Equation 4–4 puts together these two effects, calculating the change in the capital stock as $\Delta K = I - (d \times K) = \$2\text{ billion} - (0.03 \times \$30\text{ billion}) = \$1.1\text{ billion}$. Thus the capital stock increases from $30 billion to $31.1 billion. This new value of the capital stock then is inserted into the production function in equation 4–1, allowing for the calculation of a new level of output, Y.

The fifth and final equation of the model focuses on the supply of labor. To keep things simple, we assume that the labor force grows exactly as fast as the total population. Over long periods of time, this assumption is fairly accurate. If n is equal to the growth rate of both the population and the labor force, then the change in the labor force (ΔL) is represented by

$$\Delta L = nL \qquad [4\text{–}5]$$

If the labor force consists of 1 million people and the population (and labor force) is growing by 2 percent, the labor force increases annually by 20,000 (1 million × 0.02) workers. The labor force now consists of 1.02 million people, a figure that can be inserted into the production function for L to calculate the new level of output. (If we divide both sides of equation 4–5 by L, we can see directly the rate of growth of the labor force, $\Delta L/L = n$.)

These five equations represent the complete model.[5] Collectively, they can be used to examine how changes in population, saving, and investment initially affect

[5]Note that because the model has five equations and five variables (Y, K, L, I, and S) it always can be solved. In addition, there are three fixed parameters (d, s, and n), the values of which are assumed to be fixed exogenously, or outside the system.

the capital stock and labor supply and ultimately determine economic output. New saving generates additional investment, which adds to the capital stock and allows for increased output. New workers add further to the economy's capacity to increase production.

One way these five equations can be simplified slightly is to combine equations 4–2, 4–3, and 4–4. The aggregate level of saving (in equation 4–2) determines the level of investment in equation 4–3, which (together with depreciation) determines changes in the capital stock in equation 4–4. Combining these three equations gives us

$$\Delta K = sY - dK \qquad [4\text{–}6]$$

This equation states that the change in the capital stock (ΔK) is equal to saving (sY) minus depreciation (dK). This expression allows us to calculate the change in the capital stock and enter the new value directly into the aggregate production function in equation 4–1.

THE HARROD-DOMAR GROWTH MODEL

As we have stressed, the aggregate production function (shown earlier as equation 4–1) is at the heart of every model of economic growth. This function can take many different forms, depending on what we believe is the true relationship between the factors of production (K and L) and aggregate output. This relationship depends on (among other things) the mix of economic activities (for example, agriculture, heavy industry, light labor-intensive manufacturing, high-technology processes, services), the level of technology, and other factors. Indeed, much of the theoretical debate in the academic literature on economic growth is about how to best represent the aggregate production process.

THE FIXED-COEFFICIENT PRODUCTION FUNCTION

One special type of a simple production function is shown in Figure 4–1. Output in this figure is represented by **isoquants**, which are combinations of the inputs (labor and capital in this case) that produce equal amounts of output. For example, on the first (innermost) isoquant, it takes capital (plant and equipment) of $10 million and 100 workers to produce 100,000 keyboards per year (point *a*). Alternatively, on the second isoquant, $20 million of capital and 200 workers can produce 200,000 keyboards (point *b*). Only two isoquants are shown in this diagram, but a nearly infinite number of isoquants are possible, each for a different level of output.

The L-shape of the isoquants is characteristic of a particular type of production function known as **fixed-coefficient production functions**. These production