7 CHAPTER

METHODS, TOOLS AND TECHNIQUES

Chapter Outline

Learning Objectives

- Understand the terminology associated with macroeconomic modelling.
- Gain a facility for solving simple linear macroeconomic models.
- Present a macroeconomic model in graphical form.
- Construct and interpret index numbers associated with economic variables.
- Compute growth rates.
- Recognise that economic concepts and relationships can be presented in different ways, such as algebraic and graphical and through a narrative.

7.1 Overview

In macroeconomics we often deploy symbols to represent real world variables of interest, such as real GDP, consumption, and investment. In this case, while the symbols can have an abstract meaning (for example, Y is real GDP) they will also usually have a quantitative analogue (for example, for the September quarter 2013, real GDP in Australia was estimated to be $A387,031 million).

In Chapter 1, the concept of a model was introduced. A model is a generalisation about the way a system functions or behaves. It could easily be a narrative statement such as “a household will consume a proportion of their income after tax”. That theoretical statement might then be examined for its empirical relevance but will also stimulate further theoretical work trying to provide an explanation for that conjectured behaviour.

A macroeconomic model expresses our theoretical conjectures about the relationships between the main macroeconomic variables such as employment, output and inflation.

In economics, like other disciplines that use models, the narrative statement might be simplified with some mathematical statement involving symbols. In this context, the models will be represented by a number of equations (which could vary from one equation to hundreds or even thousands of equations) that describe relationships between the variables of interest. Thus, mathematics is a form of shorthand in terms of concisely representing relationships between variables. We can then apply the basic rules of algebra to conduct our analysis.

We typically use letters (such as Y) to denote different macroeconomic variables. A variable can take on different values in different time periods. The correspondence between the shorthand symbol and the variable is not always intuitive but we maintain consistent conventions throughout this textbook.

Greek symbols (such as a) are often used to denote the parameters of the model that contribute to the formalisation of the relationships between variables. In the first instance these parameters are usually assumed to be constant over time.

So Y is often used to denote real GDP or national income (but it can also be used to denote total output). C is usually used to denote final household consumption and I total private investment. X is typically used to denote exports and M imports, although in some macroeconomic textbooks M is used to denote the stock of money. We use M only to denote imports.

There are several types of variables used in macroeconomic models and the following classification is useful:

- **Endogenous** or dependent variable – its value is determined by the solution to the model and is thus dependent on the values of other variables.
- **Exogenous** or autonomous variable – its value is given in advance of solving the model.

As noted, variables are related by way of equations which express the structure of the macroeconomic model. Usually a variable that we seek to explain is written on the left-hand side of the equals sign (=) and is then expressed in terms of some other variables, placed on the right-hand side of the equals sign, that we consider are influential in explaining the value and movement of the left-hand side variable of interest.

The relationship between the variables on the right- and left-hand sides of the equation is described in terms of some **coefficients** (or parameters).

For example, $y = 2x$ is an equation which says that variable y is equal to two times variable x. The equals sign (=) tells us that the left side of the equals sign is of the same magnitude as the right side (that is, an equation has equal left and right sides).

You solve an equation by substituting values for the unknowns. The number 2 in the equation is called a coefficient and is an estimate of the way in which y is related to x.

So if $x = 1$, then we can solve for the value of $y = 2$ as a result of this equation.

A coefficient can also be called a **parameter**, which is a given in a model and might be estimated using econometric analysis (regression) or assumed by intuition so that the coefficient’s value is strictly unknown.

For example, we might have written the above equation as y = bx, where b is the unknown coefficient. You will note that we would be unable to 'solve' for the value of y in this instance even if we knew the value of x. In the case above where we said x = 1, then all we could say is that y = b. We would thus need to know what b was before we could fully solve for y.

There are several types of equations that are used in macroeconomic models.

- **Identity equation** – is an expression that is true by definition and is usually based on an accounting statement, as was noted in Chapter 6. For example, we will see that GDP is equal to the sum of the expenditure components, which is true as a result of the way we set up the national accounts and define the expenditure components.
- **Behavioural equation** – captures the hypotheses we form about how a particular variable is determined. These equations thus represent our conjectures (or theory) about how the economy works and obviously different theories will have different behavioural equations in their system of equations (that is, the economic model).
- **Equilibrium equation** – is an expression that captures a relationship between variables that defines a state of rest.

While the example of y = 2x was easy to solve once we knew the value of x, sometimes it is useful to have models which we cannot solve for numerical values of the unknown variables of interest. However, we may be able to simplify the equations to show the structure of the model in terms of what is important to advance our understanding of the relationship between our aggregates.

7.2 Basic Rules of Algebra

You will need to learn some basic algebraic rules that are used to manipulate equations and solve for unknowns of interest. Often you will need to rearrange a given equation in order to determine the solution for the variable of interest.

Model solutions

In a system of equations, the values of some variables are unknown and are only revealed when we **solve** the model for unknowns.

So if we have these two equations, which comprise a **system** of equations:

(7.1) $y = 2x$

(7.2) $x = 4$

then x is a predetermined variable (with the value of 4) and is thus exogenous. You do not know the value of y in advance and you have to solve the equations to reveal its value, so it is endogenous. To solve this system, we substitute the value of x in Equation (7.2) into Equation (7.1) so we get:

y = 2 times 4

$y = 8$

More generally the solution of a **system of structural equations** entails expressing each of the endogenous variables, say $y_1, y_2, \ldots, y_n$ as functions of the exogenous variables $x_1, x_2, \ldots, x_m$, so that there are $n + m$ 'solution' equations. In the above example, two solution equations are formed from one endogenous variable (y in Equation (7.1)) and one exogenous variable (x in Equation (7.2)).

BOX 7.1 RULES OF ALGEBRA

Addition and subtraction

In general, what we add to or subtract from one side of the equation, we have to add to or subtract from the other side to maintain the equality.

Given an equation $y = x$, then we know that the equivalent expression is $y \pm z = x \pm z$. So, for example, $y = x$ is equivalent to $y + 2 = x + 2$.

We can also substitute an expression from one equation into another and maintain equivalence.

For example, we might have $y = 2x$, and $x = 6z$. In this case, it is equivalent to write $y = 2(6z) = 12z$.

Multiplication and division

Given an equation $y = x$, then we know that the equivalent expression is $3y = 3x$ or $y/3 = x/3$. If we multiply or divide the left-hand side of the equation by a variable (or more complex algebraic expression) then we have to multiply or divide the right-hand side of the equation by the same variable (or expression). Dividing by zero is not allowed, however, and multiplying by zero is not very helpful!

Then the known values of the m exogenous variables can be substituted into each of the n equations which yield solutions for the endogenous variables, say $y_1^*, y_2^*, \ldots, y_n^*$. There are constraints on the initial system of (structural) equations for there to be a unique solution for the unknown exogenous xs and endogenous ys. These include the condition that the number of equations equals the number of unknowns, in this case $n + m$.

In modelling an economic system, it often becomes very complicated to ascertain which variables can be considered endogenous and which are truly exogenous. At the extreme, everything might be considered endogenous and then things get mathematically complex. There is a body of econometric theory which explores the problem of identifying values of coefficients in empirical work, but this topic falls well beyond the scope and aims of this textbook.

7.3 A Simple Macroeconomic Model

An example of an identity is the famous national income identity depicting aggregate demand and output, which we considered in Chapter 4:

(7.3) $Y = C + I + G + X \# M$

Recall that Y is GDP (total income and output), C is household consumption expenditure, I is private capital formation or investment expenditure, G is total government expenditure, X is total exports and M is total imports. We have used the identity sign (#) instead of the equals sign (=) to distinguish Equation (7.3) from a behavioural equation which is always expressed using an equals sign.

This identity is also an equilibrium condition in the simple national income model but it provides no information about how the right-hand side variables behave, that is, what factors influence them. To advance that understanding we form theories about the determinants of these variables which are expressed in behavioural equations. An example of a behavioural equation is the simple consumption function:

(7.4) $C = C_0 + cY_d$

which says that final household consumption (C) is equal to a constant (C_0) plus a proportion (c) of final disposable income (Y_d). The constant component (C_0) is the consumption that occurs if there is no income and can be considered to be dissaving.

Note that subscripts are often used to add information to a variable. So we append the subscript d to our income symbol Y to qualify it and denote disposable income (total income after taxes).

We also use subscripts to denote time periods when we are considering a variable over time. So Y_t indicates we are considering the value of Y at time period t. Similarly, $Yt - 1$ refers to the value of Y at time period t – 1, where the lag (the –1) depends on the periodicity of the data. If we were using quarterly data, then t – 1 would be previous quarter and so on.

In macroeconomics, some behavioural coefficients are considered important and are given special attention. So the coefficient c in the consumption function is called the marginal propensity to consume (MPC) and denotes the extra consumption per dollar of extra disposable income. So if c = 0.8 we know that for every extra dollar of disposable income that the economy produces, 80 cents will be spent on consumption.

The MPC is intrinsically related to the marginal propensity to save (MPS) which is the amount of every extra dollar of disposable income that is saved after households decide on their consumption. So $MPS = 1 - MPC$ by definition.

The importance of MPC is that it is one of the key determinants of the expenditure multiplier (see Section 7.5). We will consider this in Chapter 15 when we discuss the expenditure multiplier.

We have already introduced the distinction between exogenous variables and endogenous variables which are determined by solving the system of equations.

An exogenous variable is known in advance of 'solving' the system of equations. We take its value as given or predetermined. We might say, by way of simplification, that government spending (G) in Equation (7.3) is equal to $100 billion, which means that its value is known and not determined within the model.

The identity and behavioural equations form a macroeconomic system. This is of course a very simple system. For the sake of exposition, we might assume that the economy is closed, which means there are no exports or imports. In that case, the national income identity becomes $Y \# C + I + G$.

We also assume that there is no taxation in the model, so that disposable income is equal to total income (Y).

The model is now:

(7.5) $Y = C + I + G$

(7.6) $C = C_0 + cY$

For simplicity, we will assume that I and G are exogenous because their values are known in advance. The remaining two variables Y and C are endogenous so their values are dependent on the solution to the model.

How do we solve for Y?

We substitute (7.6) for C in (7.5) such that:

(7.7) $Y = C_0 + c_Y + I + G$

We can now rearrange this (noting we have terms in Y on both sides) by subtracting cY from both sides as per our algebraic rules. This gives:

(7.8) $Y \# c_Y = C_0 + I + G$

You will note that there are only predetermined variables (knowns) on the right-hand side now.

Because Y is what we call a common factor on the left-hand side we can write $Y - cY$ as $Y(1 - c)$.

(7.9) $Y(1 \# c) = C_0 + I + G$

Note that this has not affected the terms on the right-hand side.

We now divide both sides by $(1 - c)$, which maintains the equality between the two sides of the equation. This isolates (or solves for) Y on the left-hand side. Thus:

(7.10) $Y = [C_0 + I + G]/ (1 \# c)$

So in words, the equilibrium value of Y is a function of the autonomous variables in the model $C_0 + I + G$

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We call Equation (7.10) the **reduced form solution** of the model in which there are only exogenous or predetermined variables on the right side and an unknown variable on the left-hand side of the equation.

In a macroeconomic model, all the endogenous variables can be expressed in reduced form. So in our example, our solution for consumption would be:

(7.11) $C = C_0 + c_Y = C_0 + c[C_0 + I + G]/(1 \# c) = [C_0 + c(I + G)]/(1 \# c)$

Make sure you can derive the steps that we would take to obtain this solution.

The reduced form of the system allows us to conduct sensitivity analysis, which involves changing the values of the exogenous variables or the coefficients (in this case, the MPC) and analysing the impact on the endogenous variables, C and Y, in the model.

As an example, what would be the impact of an expansion in government spending G on national income Y? Note that we assume the other exogenous variables are unchanged.

From Equation (7.10), we know that when $G = G_0$:

(7.10a) $Y_0 = [C_0 + I + G_0]/(1 \# c) = [C_0 + I]/(1 \# c) + [G_0]/(1 \# c)$

Note that we have separated G from I and C_0.

If G rises to G_1, then:

(7.10b) $Y_1 = [C_0 + I]/(1 \# c) + [G_1]/(1 \# c)$

So, a change in Y of $(Y_1\ 2\ Y_0)$ is the difference between (7.10a) and (7.10b):

(7.10c) $(Y_1 \# Y_0) = [G_1 \# G_0]/(1\# c)$

To simplify our notation, we will usually denote the change in a variable using the Greek symbol Δ. So Equation (7.10c) would be written as:

(7.10d) $\Delta Y = \Delta G/(1 \# c)$

where the time span denoted by Δ is revealed by the context.

By dividing both sides of the equation by ΔG, we can express Equation (7.10d) as:

(7.10e) $\Delta Y/\Delta G = 1/(1 \# c)$

The right-hand side of Equation (7.10e) is known as a **multiplier** because it tells us the magnitude of the change in Y for a unit change in G. We will examine multipliers in more detail in Chapter 15, where we include taxation and exports and imports in an open economy.

7.4 Graphical Depiction of a Macroeconomic Model

We will also express our theories in graphical terms, which are an alternative to mathematical representation.

1. When income is zero, household consumption is positive. Household consumption rises proportionately with disposable income but the proportion is less than one.
2. $C = C_0 + cY_d$, where $0 < c < 1$ and C_0 is a constant (fixed value). The less than sign (<) tells us that the c (MPC) lies between the value of zero and one, that is, it is positive but less than one.

For now, we assume taxes are zero which means that national income (Y) and national disposable income (Y_d) are equal.

Considering the consumption function, if $C_0 = 100$, and $c = 0.8$, and $Y= 400$, we can solve the equation $C = C_0 + cY$ by inserting the known values of the parameters and the explanatory variable (in this case, income) into the equation and solving it. Thus:

(7.12) $C = C_0 + c_Y = 100 + 0.8 \times 400 = 420$

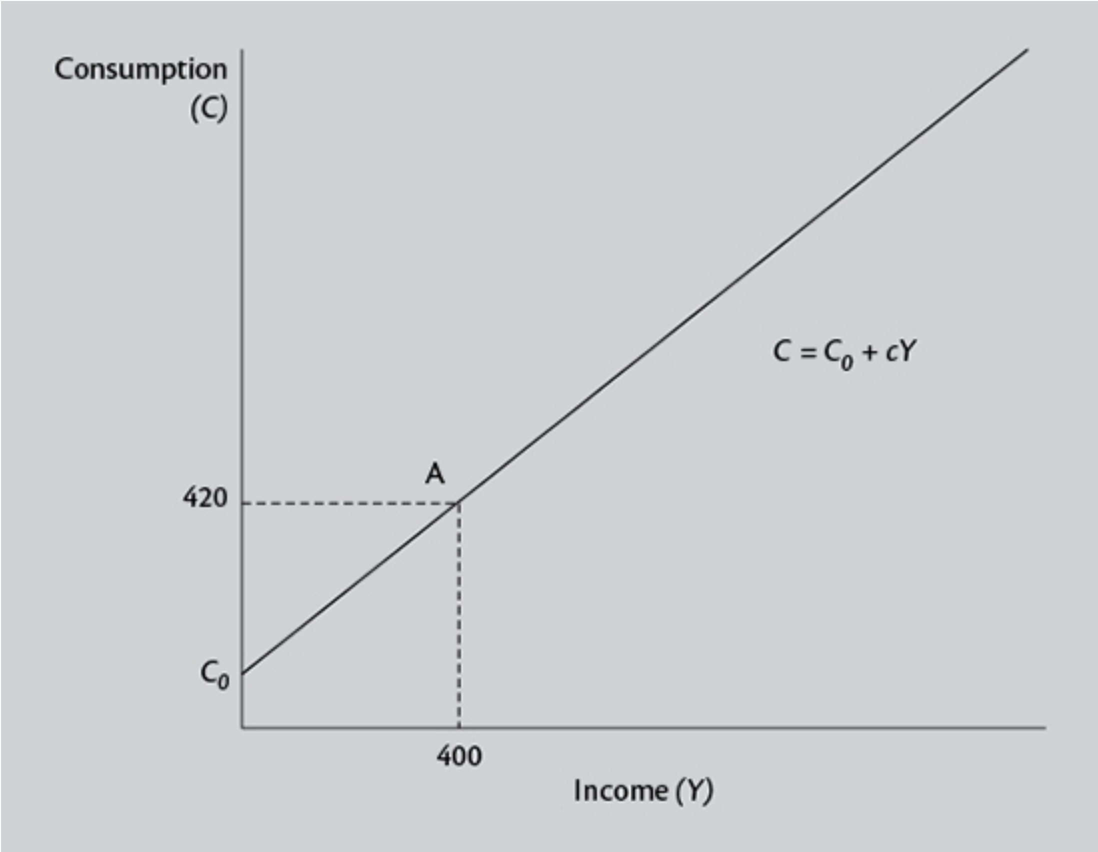
Figure 7.1 shows the consumption function for Equation (7.12). You can see that by tracing a vertical line from where disposable income equals 400 up to the graph line (A) and then a horizontal line to the vertical axis we derive the value of consumption by where that line crosses the vertical axis.

The slope of the line is the marginal propensity to consume (c). In Chapter 15 we will deal with applications of the slope of a line when we study the principle of the expenditure multiplier.

To advance our understanding of graphical methods, assume that national income rose to 1,000.

Figure 7.2 shows the combination of Y and C that we have already determined. Point A represents the combination $(Y_1, C_1) = (400, 420)$. It also shows the second combination, denoted B, which is found by the same process as point A. Trace a vertical line from where disposable income equals 1,000 up to the graph line and then a horizontal line to the vertical axis to give the value of consumption. Point B then represents the combination $(Y_2, C_2) = (1,000, 900)$.

Figure 7.1 Consumption function



We can check our graphical solution by calculation using the equation $C = C_0 + cY$ = 100 + 0.8 × 1,000 = 900.

We represent the slope of the line that intersects A and B using the following formula:

$$C = \frac{C_2 - C_1}{Y_2 - Y_1} \tag{7.13}$$

The slope of the line is given by the change in C divided by the change in Y. You can see from Figure 7.2 that this is also the ratio of the segments, BC/AC, or in words **Rise over Run**. This rule generalises to any linear function. Note that for a line with negative slope, Rise would be Fall.

You can check the slope of our consumption function by noting that BC = 480 and AC = 600 so BC /AC = 0.8, which is the MPC (c).

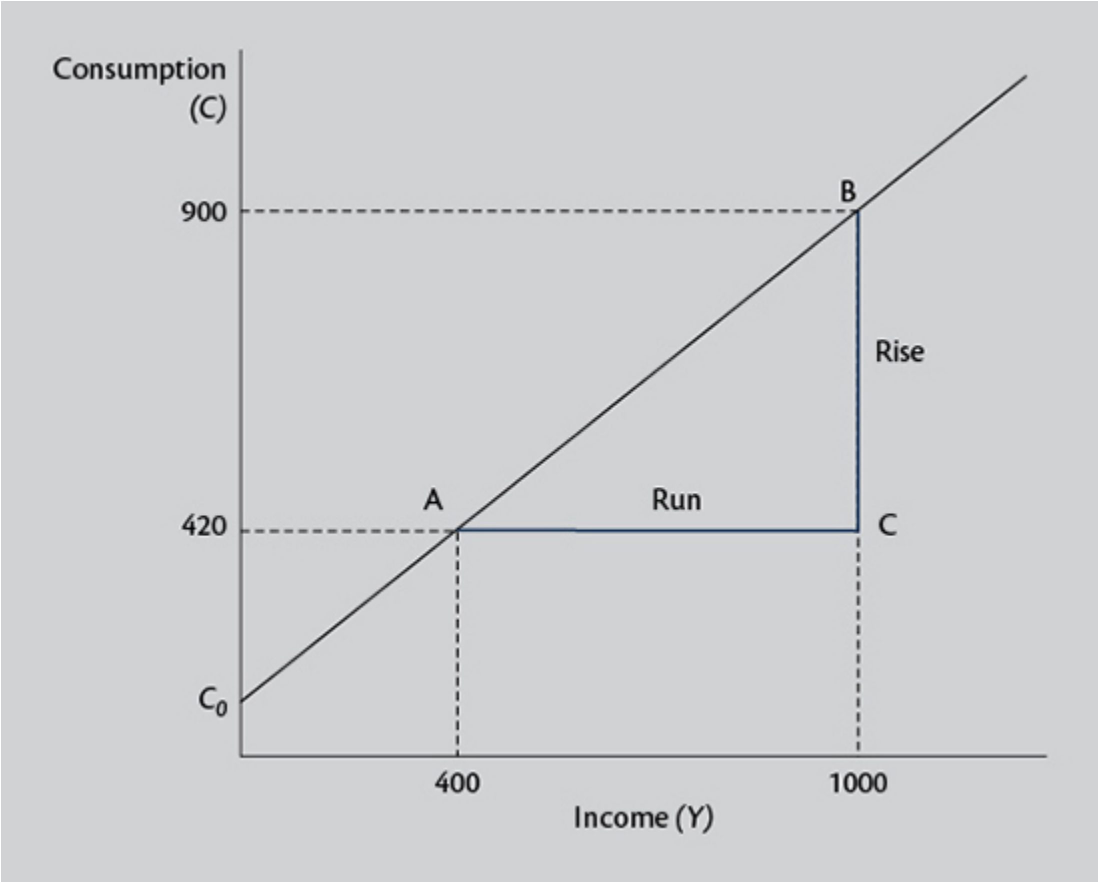
We noted earlier that we usually denote the change in a variable using the Greek symbol Δ. In that notation the slope of the consumption function would be $c = \Delta C/\Delta Y$ and that slope would be constant at all points on the (linear) function, irrespective of the magnitude of ΔY (and the corresponding value of ΔC).

The graphical approach to determining the slope of the line is confined to linear equations. If the function is non-linear (for example, a curve), the slope formula (Equation 7.13) only provides us with the average slope between two points.

A more general approach is required when the relationship of interest is non-linear and the slope is constantly varying. In this situation, differential calculus is used and the slope of the function at some specific point is defined by the **derivative** of the function.

The principle of calculating the slope from $c = \Delta C/\Delta Y$ is the same as described above. However, when deploying differential calculus, we are assuming that ΔY is infinitesimally small (that is, a value approaching zero), as is ΔC. Otherwise, in contrast to a linear function, there will be ambiguity in measuring the slope because it will depend on the chosen magnitude of ΔY, as well as the value of Y at which it is measured. Typically, we will be concerned with changes that are not infinitesimally small and so the tools of calculus will be of limited use to us in this textbook. Thus, we will use calculus sparingly but it is useful to understand the basic concept of a derivative.

Figure 7.2 Slope of consumption function



The slope of a function $y = f(x)$ is $\Delta y/\Delta x$. We can see that $\Delta y = f(x + \Delta x) - f(x)$. Now what would happen if Δx was close to zero?

Take a specific example of a non-linear function, $y = x^2$.

If $f(x) = x^2$, $f(x + \Delta x) = (x + \Delta x)^2 = x^2 + 2x\,\Delta x + (\Delta x)^2$

Note that we do not expect students to be able to derive these expressions themselves. Rather, we hope you can appreciate the concept being outlined.

Now if we substitute this into the slope formula we get:

$$\text{If } f(x) = x^2, f(x + \Delta x) = (x + \Delta x)^2 = x^2 + 2x\,\Delta x + (\Delta x)^2$$

The derivative of a function $y = f(x)$ is the limiting value of $\Delta y/\Delta x$ as Δx # 0.

So from Equation (7.14), the derivative of $y = x^2$ is $2x$, since Δx # 0 and can be ignored. Thus the derivative of a non-linear function of x depends on the chosen value of x. The derivative of a function $y = f(x)$ is written as dy/dx or $f'(x)$.

What is meant by the derivative of a function? It means that if $y = x^2$, the rate of change of the value of the function, y, at any point is $2x$. If $x = 4$, then the rate of change in y is 8. If $x = 5$, then the rate of change in y is 10.

More generally, if $y = x^n$, it can be shown that $dy/dx = nx^{n-1}$. Thus, if $y = x^3$, $dy/dx = 3x^2$, which can be evaluated at any value of x.

For the consumption function, $C = C_0 + cY$, applying the general formula for a derivative would yield $dC/dY = cY^{1-1} = cY^0 = c$, which is consistent with what we have already found. Note that $Y^0 = 1$.

7.5 Power Series Algebra and the Expenditure Multiplier

We have introduced the concept of the expenditure multiplier, which we consider further in Chapter 15. It allows us to calculate the total change in national income following a change in one of the autonomous components of aggregate demand, such as government spending or private investment.

Multipliers play a central role in discussions of the economic impact of policy interventions. The multiplier depends on the magnitude of the initial injection of expenditure plus the induced expenditure that follows. We will learn that when, for example, the government increases its spending, national income rises by that amount, which in turn induces further consumption expenditure.

Thus, if one dollar was injected into the economy, through additional spending, total income would initially rise by one dollar. If the marginal propensity to consume was 0.8, then this initial rise in income would induce a rise in consumption of 0.8 3 $1 or 80 cents in period 1. This initial $0.80 rise in induced spending would further induce a rise in consumption in period 2 of 0.8 3 0.8 or 64 cents and so on. Note that this is a simple, mathematical exposition of what in the real world would be a complex process of adjustment of the economy to an increase of government spending.

The multiplier is thus related to the slope of the function and the algebraic notion of a geometric or power series.

Consider a one dollar increase in government spending with c representing the marginal propensity to consume and n being the number of periods. Then we could write the multiplier over n periods as $k(n)$, where:

(7.15) $k(n) = 1 + c + c^2 + c^3 + c^4 + \ldots + c^n$

The right-hand expression is called a **power series** with each term being a constant multiple of the previous term.

Note that multiplying both sides of (7.15) by c gives:

(7.16) $ck(n) = c + c^2 + c^3 + c^4 + c^5 + \ldots + c^n + c^{n11}$

If we subtract (7.16) from (7.15) we get:

(7.17) $k(n) \# ck(n) = k(n)(1 \# c) = 1 \# c^{n+1}$

which gives:

(7.18) $k(n) = (1 \# c^{n+1})/(1 \# c)$

For a large number of periods we consider $n\#$、 so the value of c^{n+1} tends to zero because $0 < c < 1$. We denote the summation by k^*. Hence:

(7.19) $k^* = 1/(1 \# c)$

Equation (7.19) shows that the multiplier, k^*, is $1/(1 - c)$, where c is the marginal propensity to consume, when n is a large number. It should be interpreted as the overall impact of the multiplier process, rather than considering the impact over a finite number of periods.

7.6 Index Numbers

An index number allows the comparison between the values of a variable over time. For example in Chapter 4, the construction of a consumer price index (CPI) was described. However, if two or more variables are expressed in index number form, then straightforward comparisons of their respective rates of change (growth rates) can be made.

The creation of an index number requires a starting point (the base period or value) which is usually set at 100. Each observation is then expressed as a percentage of the base year. For example, consider the data in Table 7.1, which shows full-time, part-time and total employment for Australia from 2000 to 2012. The data are reported in units of thousands.

Visual inspection of the data can provide information as to the evolution of employment over this time period in Australia. But what if we wanted to know whether full-time employment had grown more quickly or more slowly than part-time employment since 2000? This is where index numbers are useful.

To convert this data into index numbers we first define the base as the year 2000 and set that index value to 100. We then generate index values of full-time employment for 2001 through to 2012, by comparing employment in each of these years with employment in 2000 (for which the index value is 100). To do this we divide each value of full-time employment by its value in 2000 (6,614.6) and then multiply by 100. The index number for full-time employment in 2001 would thus be 100 × (6,597.5/6, 614.6) = 99.7 and so on.

Table 7.2 shows the index numbers corresponding to the three employment time series. Note that while the raw employment data is expressed in units of thousands, the index numbers are unit free. Thus defining a common base year for the three series (that is, values in 2000 are set at 100) means that comparisons between them can be readily made, even though their corresponding absolute values differ quite significantly.

Table 7.1 Employment in Australia, 2000–12

	Full-time employment (000s)	Part-time employment (000s)	Total employment (000s)
2000	6,614.6	2,375.1	8,989.7
2001	6,597.5	2,492.4	9,089.9
2002	6,648.7	2,622.0	9,270.7
2003	6,772.7	27.12.2	9,485.0
2004	6,930.4	2,730.9	9,661.3
2005	7,148.7	2,849.4	9,998.1
2006	7,326.9	2,930.2	10,257.1
2007	7,583.7	2,993.3	10,577.0
2008	7,782.5	3,093.1	10,875.6
2009	7,724.3	3,229.5	10,953.7
2010	7,852.5	3,337.6	11,190.0
2011	8,014.5	3,376.1	11,390.6
2012	8,098.3	3,417.1	11,515.4

Source: Data from Australian Bureau of Statistics, Labour Force.

Table 7.2 Employment indices for Australia, 2000–12

	Numbers in employment			Employment index		
	Full-time employment (000s)	Part-time employment (000s)	Total employment (000s)	Full-time employment (2000=100)	Part-time employment (2000=100)	Total employment (2000=100)
2000	6,614.6	2,375.1	8,989.7	100.0	100.0	100.0
2001	6,597.5	2,492.4	9,089.9	99.7	104.9	101.1
2002	6,648.7	2,622.0	9,270.7	100.5	110.4	103.1
2003	6,772.7	27.12.2	9,485.0	102.4	114.2	105.5
2004	6,930.4	2,730.9	9,661.3	104.8	115.0	107.5
2005	7,148.7	2,849.4	9,998.1	108.1	120.0	111.2
2006	7,326.9	2,930.2	10,257.1	110.8	123.4	114.1
2007	7,583.7	2,993.3	10,577.0	114.7	126.0	117.7
2008	7,782.5	3,093.1	10,875.6	117.7	130.2	121.0
2009	7,724.3	3,229.5	10,953.7	116.8	136.0	121.8
2010	7,852.5	3,337.6	11,190.0	118.7	140.5	124.5
2011	8,014.5	3,376.1	11,390.6	121.2	142.1	126.7
2012	8,098.3	3,417.1	11,515.4	122.4	143.9	128.1

We can see that the index number for full-time employment rose from 100 in 2000 to 122.4 in 2012, a 22.4 per cent increase. In the same period the part-time employment index number rose from 100 in 2000 to 143.9 in 2012, a 43.9 per cent rise which is almost twice as large as the increase in full-time employment.

Note that for any pair of index number observations we can also compute percentage changes. For example, what was the growth of full-time employment in 2008 and 2009?

We know that full-time employment was 7,782.5 in 2008 and had fallen to 7,724.3 in 2009. We could calculate a simple percentage 100 × (7,724.3 – 7,782.5)/7,724.3 = –0.75 per cent. We could find the same result using the index numbers 117.7 in 2008 and 116.8 in 2009 and computing 100 × (116.8 – 117.7)/117.7 = –0.76 per cent. (The small difference here is accounted for by rounding up the index numbers to one figure after the decimal point.)

It is also useful to graph index numbers if we are interested in a visual comparison of the behaviour of different variables.

For example, Figure 7.3 allows one to see easily that in 2008, when the financial crisis emerged, full-time employment in Australia contracted while part-time employment accelerated. This observation would motivate a researcher to investigate the labour market processes that underpinned this outcome.

As mentioned above, index numbers also allow us to compare the evolution of two or more variables over time when the underlying units of measurement of each variable are different. For example, for decades real wages in most nations grew in line with labour productivity, which created the space for wages to grow without invoking inflationary pressures. This observation means that economists are often interested in examining the relationship between the two variables over time.

Figure 7.4 shows the relationship between real wages and productivity growth in Australia from 1978 to 2015 in index number form. The base period is March 1982 and the underlying data are quarterly. Productivity is measured in terms of units of real GDP per hour worked while real wages are computed as the nominal wage series ($ per hour) deflated by the consumer price index.

By converting the different series to common index numbers we are readily able to see the comparative behaviour of these related time series variables.

Figure 7.3 Employment index numbers, Australia, 2000–15, January 2000=100

Source: Authors' own. Data from Australian Bureau of Statistics, National Accounts.

Figure 7.4 Real wages and productivity, 1978–2015, March quarter, 1982=100

Source: Authors' own. Data from Australian Bureau of Statistics, National Accounts.

7.7 Annual Average Growth Rates

In Chapter 4 we computed the percentage change in the CPI from one period to the next. We were essentially solving the equation:

$$CPI_t = CPI_{t21} \times (1 + r^*/100)$$

where r^* is the rate of change of the CPI, expressed as a percentage.

Economists are often required to compute how fast the economy (or some other aggregate) is growing on average over a number of years. We are then calculating a constant compound growth rate.

7.8 Textbook Policy Regarding Formalism

We recognise that students have different ways in which they learn and develop their understanding of concepts. Some prefer the mathematical approach while others prefer the graphical approach. Others still learn better through reading the written word, even though that form of communication is prone to interpretative issues.

All the essential material in the text will therefore be presented in all three ways. Sometimes the mathematical treatment will appear in the Appendix of the relevant chapter (usually the more difficult, advanced material), and sometimes within the main body of the text.

We always aim to promote understanding and believe that a student is entitled to learn in the way that best suits their own proclivities. However, it is also the case that professional economists use a variety of methods including numerical, graphical, algebraic and narrative in their work, and we believe it is important to expose students to the broad range of presentation methods deployed in the real world.

TRY IT YOURSELF

In 1960, real GDP in Australia was $249,083 million and it had grown to $1,508,267 million by 2012. To calculate the average annual compound growth rate over this 52-year period, we need to utilise some simple algebra and deploy the notion of a compound growth rate.

We can write:

$$Y_t = Y_0 (1 + r)^t$$

where Y_t is real GDP in 2012; Y_0 is real GDP in 1960; r is the average compound growth rate, expressed as a decimal, that is, $r = r^*/100$; and t is the number of periods over which we are compounding, in this case 52 years.

Here we need to use the natural logarithm (*ln*) to simplify our task.

The task is to solve for the unknown r:

$$(Y_t/Y_0) = (1 + r)^t$$

$$\ln(Y_t/Y_0) = t \times \ln(1 + r)$$

$$\frac{\ln(Y_t/Y_0)}{t} = \ln(1 + r)$$

$$exp\left[\frac{\ln(Y_t/Y_0)}{t}\right] = 1 + r$$

$$r = exp\left[\frac{\ln(Y_t/Y_0)}{t}\right] - 1$$

Table 7.3 shows the steps in the calculation and you can paste the data into a spreadsheet to derive the same results. The calculations show that the annual average compound growth real GDP growth rate for Australia between 1960 and 2012 was 3.52 per cent.

You can use the formula for any period or data frequency (for example, month, quarter, year) by substituting the appropriate information into the calculation.

Table 7.3 Compound growth rate calculations

Real GDP 1960 $m	Y_0	**249,083**
Real GDP 2012 $m	Y_t	**1,508,267**
Time periods (years)	t	**52**
GDP2012/GDP1960 (Y_t/Y_0)	$= (1 + r)^t$	**6.0553**
Take log	$= t \ln(1 + r)$	**1.8009**
Divide by 52	$= \ln(1 + r)$	**0.0346**
Take antilog	$= 1 + r$	**1.0352**
	r	**0.0352**
	r **per cent**	**3.5240**

Conclusion

This chapter provided a review of the basic mathematics that is often used in macroeconomics, including algebra, power series, index numbers, and growth rates. A simple macroeconomic model was presented to provide an example of the methodology often used. Our policy regarding formalism in this textbook was clarified: we use three methods to present all the essential material: words, graphs, and mathematics. Each student can choose the method best suited to their learning style.

Visit the companion website at www.macmillanihe.com/mitchell-macro for additional resources including author videos, an instructor's manual, worked examples, tutorial questions, additional references, the data sets used in constructing various graphs in the text, and more.