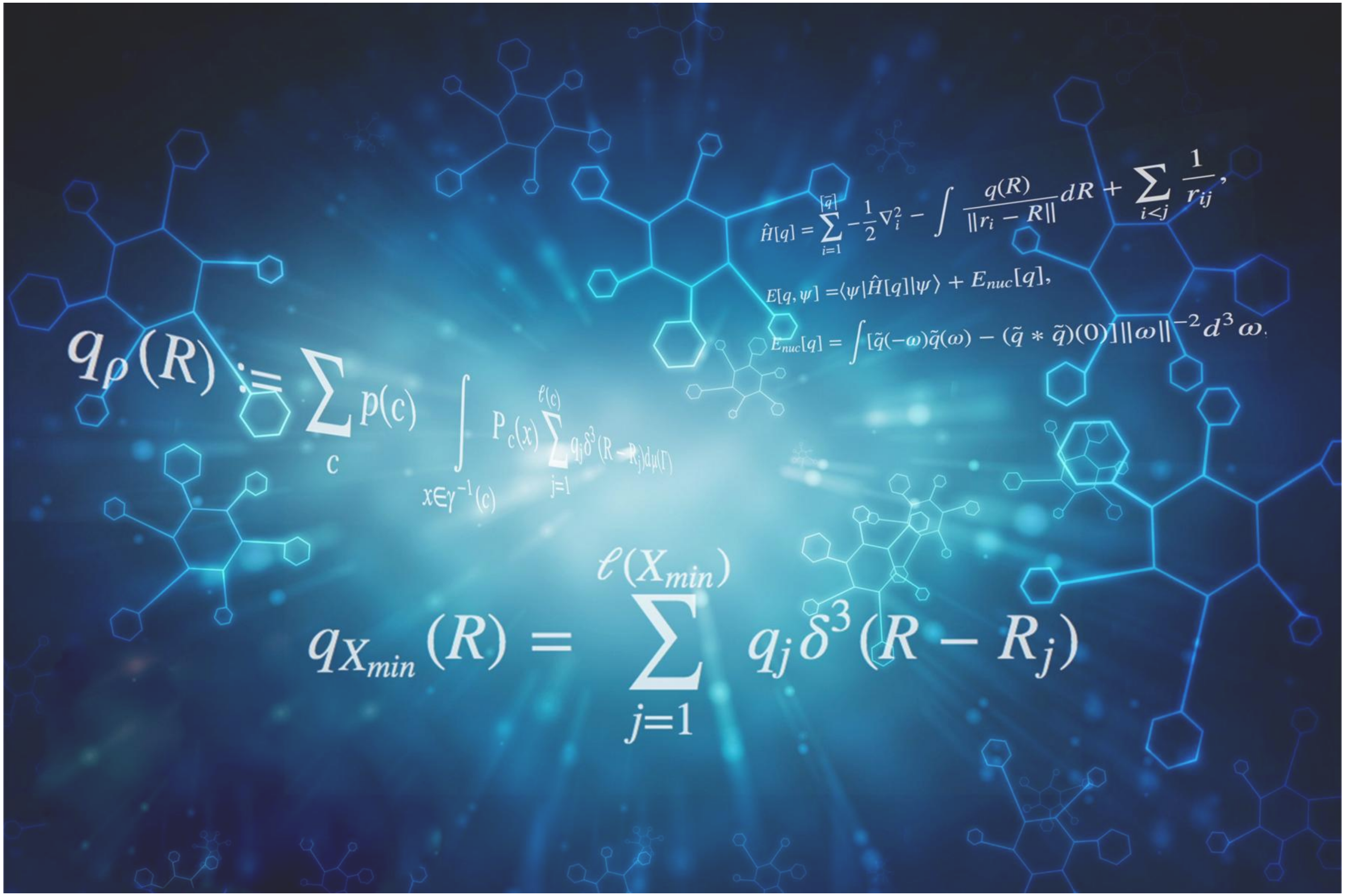


General Chemistry I, CHEM 103

Lecture 1: Measurements, mathematical treatment of measurements and dimensional analysis I

(aka the math you need to know for Gen Chem)

Chapter 1 from textbook

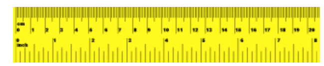
Units 1.4-1.6

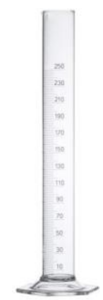
Math you need to know in Chem 103

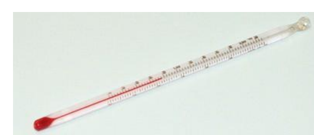
- Metric system and unit conversion
- Significant figures
- Scientific notation and exponents
- Dimensional analysis

Metric system: base and derived units (SI units)

Base Units		
Length	meter	m
Mass	kilogram gram	kg g
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Amount of substance	mole	mol
Luminous intensity	candela	cd
Some Derived Units		
Area	square meter	m^2
Volume	cubic meter liter	m^3 L (= dm^3)







Metric system: prefixes

Most commonly used in genchem (memorize these):

Kilo	k	10^{3}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	µ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}

List of Prefixes in the Metric System

Prefix	Symbol	Numerical Value	Exponential Value
quetta	Q	1,000,000,000,000,000,000,000,000,000,000	10^{30}
ronna	R	1,000,000,000,000,000,000,000,000,000	10^{27}
yotta	Y	1,000,000,000,000,000,000,000,000	10^{24}
zetta	Z	1,000,000,000,000,000,000,000	10^{21}
exa	E	1,000,000,000,000,000,000	10^{18}
peta	P	1,000,000,000,000,000	10^{15}
tera	T	1,000,000,000,000	10^{12}
giga	G	1,000,000,000	10^{9}
mega	M	1,000,000	10^{6}
kilo	k	1,000	10^{3}
hecto	h	100	10^{2}
deka	da	10	10^{1}
no prefix		1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}
femto	f	0.000000000000001	10^{-15}
atto	a	0.000000000000000001	10^{-18}
zepto	z	0.000000000000000000001	10^{-21}
yocto	y	0.000000000000000000000001	10^{-24}
ronto	r	0.000000000000000000000000001	10^{-27}
quecto	q	0.000000000000000000000000000001	10^{-30}

Practice unit conversions

- The length of the bond between two hydrogen atoms in a H_2 molecule is 74 pm. How much is that in meters?
- A wavelength corresponding to blue light is 475 nm. How much is that in cm?
- A drop of water has a volume of 50 µL. How much is that in mL?

Metric system/SI units practice

Give the derived SI units for each of the following quantities in base SI units:

- Acceleration = distance/time^2
- Force = (mass)(acceleration)
- Work =(force)(distance)
- Pressure = force/area
- Power = work/time
- Velocity = distance/time
- Energy = (mass)(velocity)^2

Some useful exponent rules

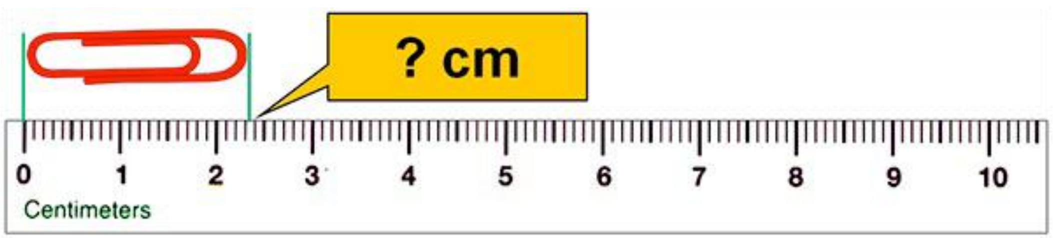
For $\mathrm{a} > 0,\ \mathrm{b} > 0,\ x,\ y,\ and\, n\ Real\ Numbers$

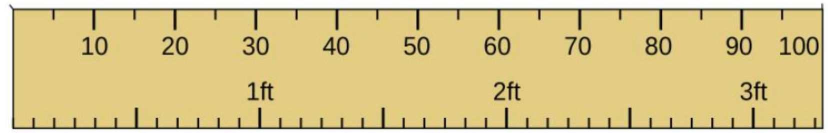
1. $\mathrm{a}^x = \mathrm{a}^y \iff x = y$
2. $\mathrm{a}^x\,\mathrm{a}^y = \mathrm{a}^{x+y}$
3. $\frac{\mathrm{a}^x}{\mathrm{a}^y} = \mathrm{a}^{x-y}$
4. $\mathrm{a}^{-x} = \frac{1}{\mathrm{a}^x}$
5. $(\mathrm{a}^x)^y = \mathrm{a}^{xy}$
6. $\mathrm{a}^x\,\mathrm{b}^x = (\mathrm{ab})^x$
7. $\mathrm{a}^{\frac{x}{y}} = \sqrt[y]{\mathrm{a}^x} = (\sqrt[y]{\mathrm{a}})^x$ for x and y integers > 1
8. $\mathrm{a}^0 = 1$
9. $\mathrm{a}^1 = \mathrm{a}$
10. $1^x = 1$

Exact vs. inexact numbers

- Exact numbers
  - Counted objects (so we know the number exactly by counting)
    - 5 beakers
  - Conversion factors that are exact by definition
    - Metric multiples and submultiples of a unit (e.g., 1 m = 100 cm)
    - 1 in = exactly 2.54 cm
  - Many physical constants
    - Avogadro constant, ideal gas constant, Planck constant, speed of light…

- Inexact numbers
  - Numbers obtained through measurement (using human made tools that by their nature have a precision limit) directly or indirectly





Significant figures

- Significant figures are the digits in a number that carry meaningful information about its precision. In science and engineering, no measurement is perfect. Significant figures prevent you from claiming your data is more precise than it actually is.
- Are these numbers the same?

  0.35 g vs. 0.3500 g

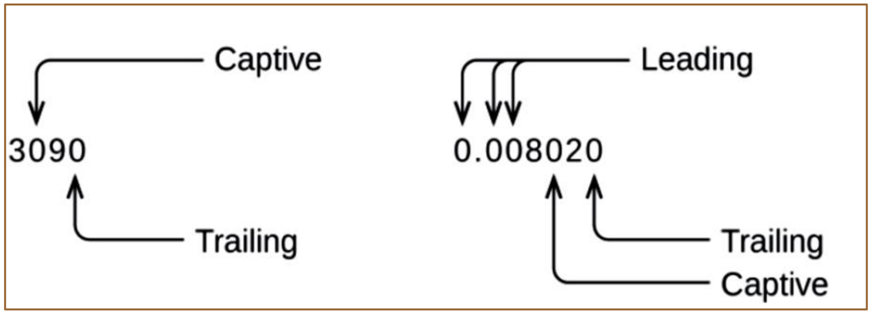
- No – 0.3500 g is 100x more precisely known!
  - 0.35 g: uncertain about the hundredths (10^{-2}) position; certain about all positions > 10^{-2}
- 0.3500 g: uncertain about the 10^{-4} position
- The value conveys knowledge about the instrument with which it was measured

$200

$5000

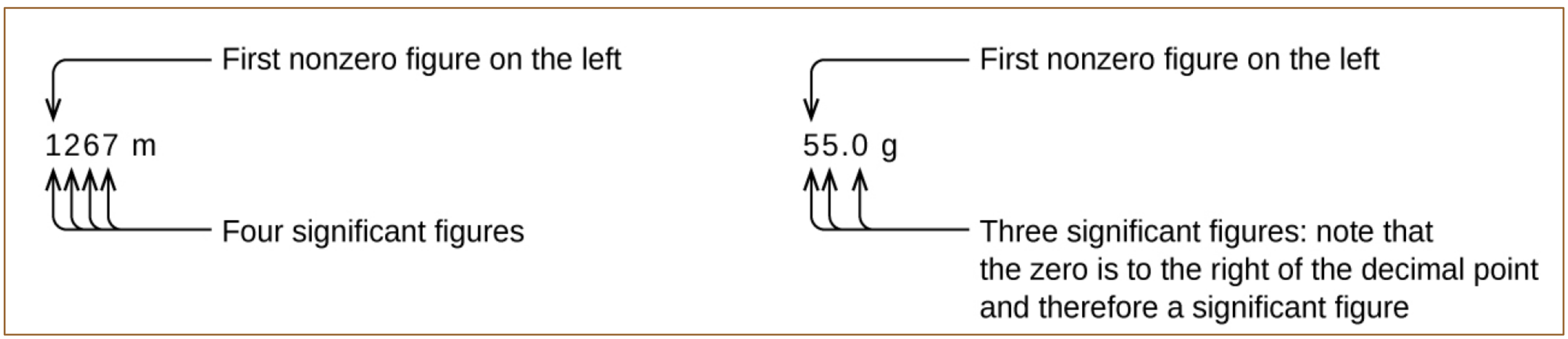
Significant figure rules

- All exact numbers have infinite significant figures
- Rest: all figures are significant, except for leading zeros and trailing zeros when there is no decimal point
  - 45.503
  - 0.406
  - 0.400
  - 0.0047
- Certain ambiguities exist, some of which can only be addressed by scientific notation
  - 7500 m
  - 7500. m
  - 7.50×10^3 m
- When in doubt – convert number to scientific notation
  - Perhaps the easiest way to see the # of SF!



Significant figure rules

1. Non-zero digits are always significant
2. Zeros between non-zero digits are always significant
3. Leading zeros are never significant
4. Trailing zeros are only significant if the number contains a decimal point



Significant figure rules: Let's practice together!

1. Non-zero digits are always significant
2. Zeros between non-zero digits are always significant
3. Leading zeros are never significant
4. Trailing zeros are only significant if the number contains a decimal point

Go to Moodle page, under today's lecture:

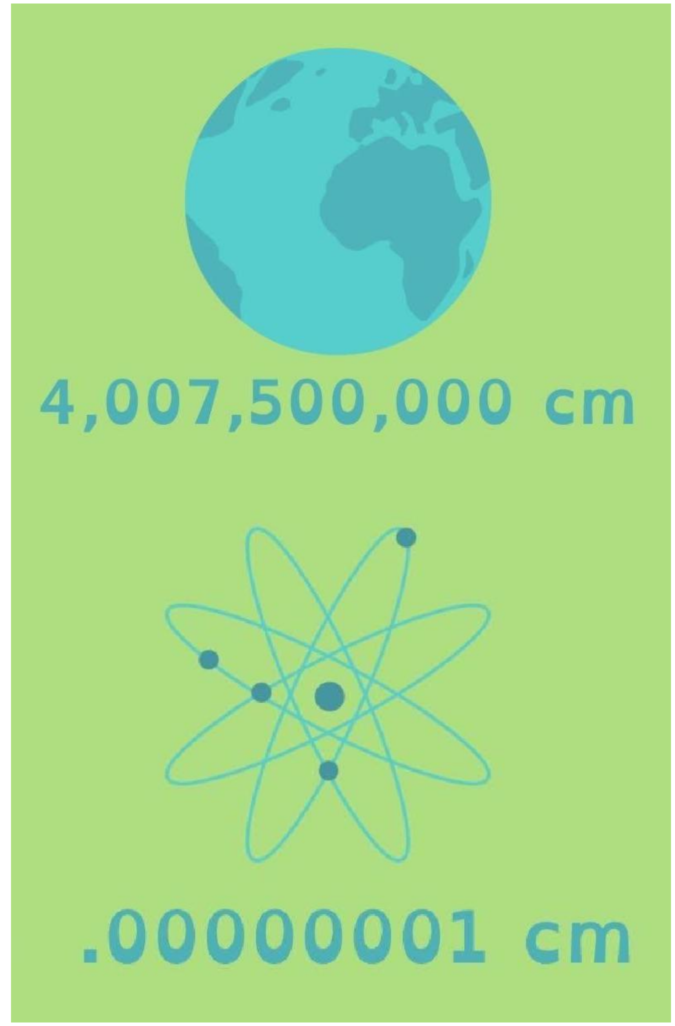
https://docs.google.com/forms/d/e/1FAIpQLSe40UW2-tjVPn_q2wl4cvjR-3D7UWjdPCt5wnzF26TuJuKemg/viewform?usp=header

Find Significant Figures

81	
26.2	
0.007	
5200.38	
380.0	
78800	
78800.	

Scientific Notation

- Best way to:
  - express very big or very small numbers
  - emphasize significant figures
- Format: a.bcd… x 10^x
- 123,000,000 = 1.23 x 10^8
- 0.0004567 = 4.567 x 10^{-4}

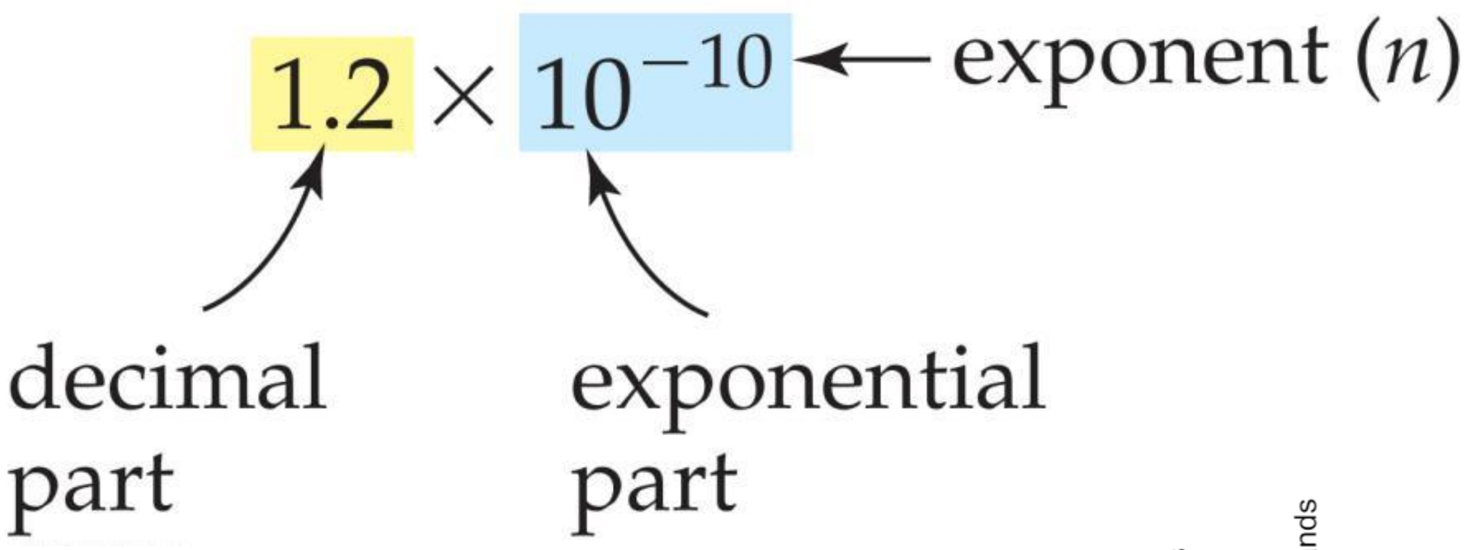


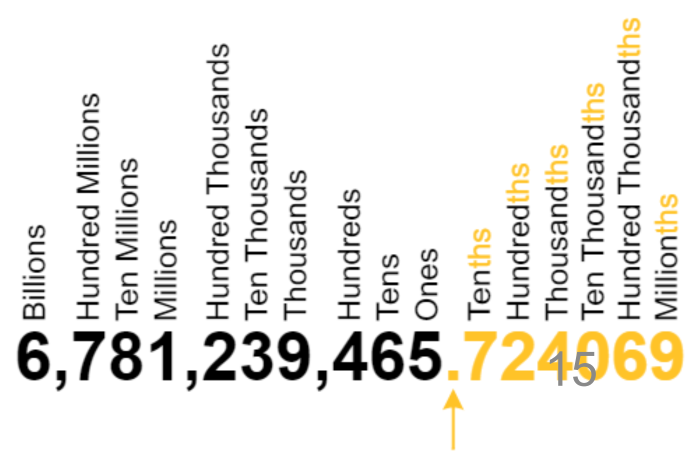
Scientific Notation

A number written in scientific notation has two parts:

The **decimal part: a number that is between 1 and 10.**

The **exponential part: 10 raised to an exponent, *n*.**





Writing in Scientific Notation

A positive exponent means 1 multiplied by 10 *n* times.

A negative exponent (–*n*) means 1 divided by 10 *n* times.

$$10^0 = 1$$

$$10^1 = 1 \times 10 = 10$$

$$10^2 = 1 \times 10 \times 10 = 100$$

$$10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

$$10^{-1} = \frac{1}{10} = 0.1$$

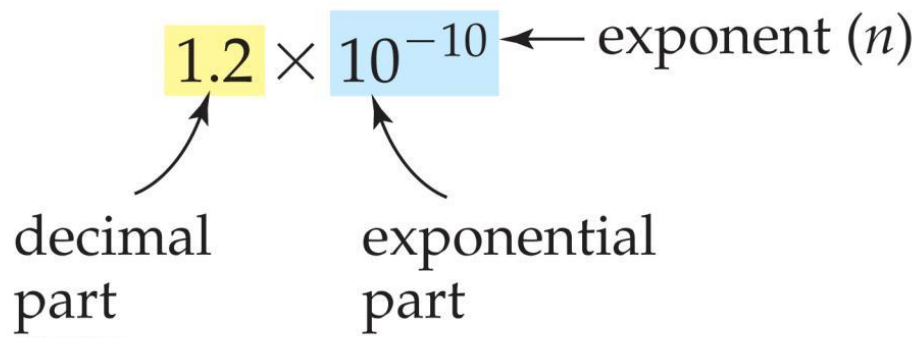
$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$

Converting a Number to Scientific Notation

Find the decimal part. Find the exponent.

0.00000000012



Move the decimal point *n* times to obtain a number between 1 and 10 (the decimal part).

Multiply the decimal part by 10^n or 10^{-n} as appropriate.

Converting a Number to Scientific Notation

$$5983 =$$

$$3\ 2\ 1$$

$$0.00034$$

$$1\ 2\ 3\ 4$$

If the decimal point is moved to the left, the exponent (n) is positive.

If the decimal point is moved to the right, the exponent (n) is negative.

Operations with significant figures: three basic rules

- Addition/subtraction
  - *Round the result to the uncertain order of magnitude of the quantity in which uncertainty begins at the highest order of magnitude.*
  - *(More typical, but less comprehensive way the rule is usually expressed): round the result to the last decimal place of the quantity with fewest decimal places.*

- Multiplication/division
  - *The number of significant figures in the answer is the same as the number of significant figures in the least precisely known number used in the calculation.*
  - Example: 34.0 g of sugar are dissolved in water to make 100.0 mL of solution. What is the solution's concentration in g/mL?

- Conversion factors used in calculations should be chosen so as not to limit precision
  - Should have more significant figures than rest of values
  - 1 lb = 453.592 g

Problem: What is the difference in mass between a nickel with mass 4.6837 g and a nickel with mass 4.6 g?

Addition and Subtraction: *Round the result to the decimal place of the least precisely known quantity (to the last common s.f.).*

significant for both numbers

significant for 4.6837 but not 4.6

$$
\begin{array}{rl}
4.6837\text{g} & \leftarrow \; 10^{-4} \text{ is uncertain} \\
-4.6\phantom{837\text{g}} & \leftarrow \; 10^{-1} \text{ is uncertain} \\
\hline
0.0837\text{g} & \leftarrow \; \text{round off to } 10^{-1}
\end{array}
$$

the last common s.f.

More than one operation? Follow the order of operations!

- Apply the appropriate SF rule as you perform each operation
- Underline last SF in each result to keep track of what you are doing
- Don’t round (in your calculator or Excel spreadsheet) until the end to avoid roundoff error!
- Example

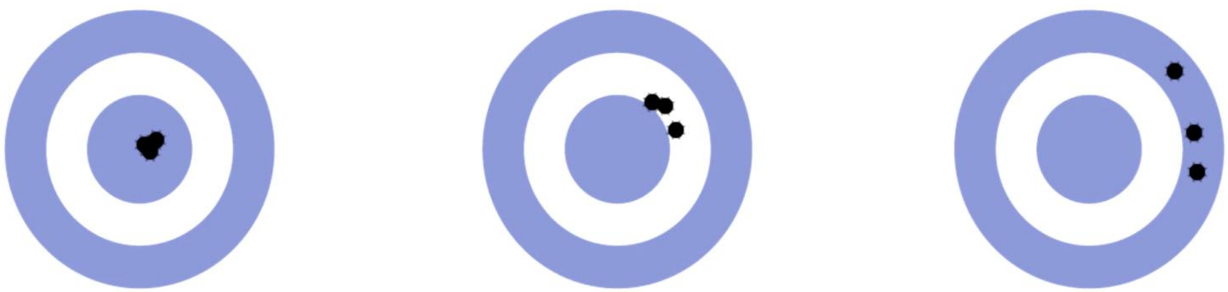
  $(1.646 \times 10^{3}) - (2.18 \times 10^{2}) + (1.36 \times 10^{4}) \times (5.17 \times 10^{-2})$

Point of confusion: significant figures and decimal places

- Both SF and DP inform us about precision (generally, the more SF/DP, the higher the precision)
- …but they do it in different ways
  - SF are affected by multiplication/division
  - DP are affected by addition/subtraction
- …and are independent of each other
  - 0.0403 g
    - 3 SF
    - 4 DP
- Don't mix them up!

Accuracy and Precision

- Accuracy = agreement with “recognized” value
- Precision = reproducibility, knowing a value to many significant figures

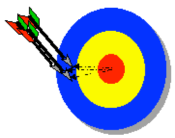


Accurate and precise (a) | Precise, not accurate (b) | Not accurate, not precise (c)

(a) These arrows are close to both the bull’s eye and one another, so they are both accurate and precise. (b) These arrows are close to one another but not on target, so they are precise but not accurate. (c) These arrows are neither on target nor close to one another, so they are neither accurate nor precise.

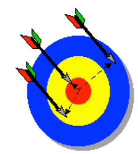
Additional uncertainty: systematic and random errors

Error



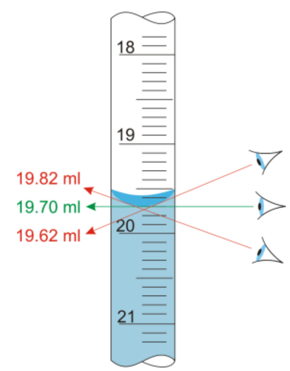
Systematic Error

Example:
inaccurately calibrated or not "zeroed" instrument



Random Error

Example:
technical errors (e.g., parallax error)



Dimensional analysis and unit conversion

- First, some definitions
- Dimension = physical quantity
  - Length, temperature, energy…
- For each dimension, there can be multiple units used to measure it
  - Some units used to measure energy (SI unit bolded): **joules (J)**, calories (cal), electron-volts (eV)…
  - Length: **meters (m)**, centimeters (cm), inches, feet…
- Dimensional analysis: process of converting from one dimension or unit to another

How to perform dimensional analysis

How much volume, in L, is occupied by 1.32 kg of mercury (Hg)? (density = 13.56 g/cm^3)

- Put units on every number!
- Identify the unit/dimension you need to change (original • final)
- List all appropriate unit conversion factors and formulas that allow dimensional analysis
  - Recognize that if a = b, then $\frac{?}{?} = \frac{?}{?} = 1$ and multiplying by 1 doesn't change the number's value
  - A value divided by itself is canceled (becomes a factor of 1)
- Arrange factors appropriately so that all unwanted units and dimensions cancel
- Calculate (and check SF)

Practice problem

- A 2.67 g piece of gold with density = 19.30 g/cm^3 is hammered into a sheet whose area is 41.9 ft^2. What is the height of the sheet in µm? (2.54 cm = 1 inch, 12 in = 1 ft)