The Tunnel of Eupalinus and the Tunnel Problem of Hero of Alexandria

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The Tunnel of Eupalinus and the Tunnel Problem of Hero of Alexandria

*By Alfred Burns**

I

THE FOLLOWING DISCUSSION of the Tunnel of Eupalinus on the Greek island of Samos relies heavily on Ernst Fabricius' original description,[1] the excellent photographs and descriptions by June Goodfield and Stephen Toulmin,[2] Wolfgang Kastenbein's meticulous survey,[3] and to some extent on my own investigations on the site. In addition to the modern descriptions, it is also important to keep Herodotus' account in mind:

> I have dwelt rather long on the history of the Samians because theirs are the three greatest works (*ergasmata*) of all the Greeks. One is a tunnel (*orygma amphistomon*) through the base of a nine hundred foot high mountain. The tunnel's length is seven stades, its height and length both eight feet. Throughout its length another cutting (*orygma*) has been dug (*orōryktai*) three feet wide and three feet deep, through which the water flowing in pipes is led into the city from an abundant spring. The builder (*architektōn*) of the tunnel was the Megarian Eupalinus, son of Naustrophus. This is one of the three, the second is a jetty around the harbor in twenty fathoms of water more than two stades long. Their third work is the largest of all temples we have seen. Its original architect was Rhoecus son of Phileus. Because of these accomplishments I have carried my account of the Samians to greater length.[4]

The admiration expressed by Herodotus, the good preservation of the tunnel, the exact confirmation of Herodotus' description, and, perhaps most of all, the surprise discovery of the tunnel's two-way construction have made it a fascinating subject of

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[1] Ernst Fabricius, "Altertümer auf der Insel Samos," *Mitteilungen des Deutschen Archäologischen Institutes in Athen*, 1884, *9*:165–192.

[2] June Goodfield and Stephen Toulmin, "How Was the Tunnel of Eupalinus Aligned?" *Isis*, 1965, *56*:45–56.

[3] Wolfgang Kastenbein, "Untersuchungen am Stollen des Eupalinos," *Archäologischer Anzeiger*, 1960, *75*:178–198.

[4] Herodotus, *Histories* 3.60 (the translation is mine).

speculation in the ninety years since its rediscovery. Most of the tunnel's problems remain unsolved:

1. What mathematical methods were used to plan direction and slope?
2. Once determined, how was the plan executed?
3. Why were the pipes laid in a special secondary tunnel rather than simply on the main tunnel's floor?
4. Does construction of the tunnel shed any light on the status of mathematics in Samos during the late sixth century B.C., and thus possibly on Pythagoras' connection with mathematics?
5. Hero of Alexandria, writing about six hundred years later on practical mathematical problems, offers a solution to the problem of how "to dig through a mountain in a straight line when the mouths of the tunnel on the mountain are given."[5] Does Hero's problem refer to our tunnel?

II

Herodotus tells us nothing about how the tunnel was built and no other specific mention of the tunnel has come down to us from antiquity. Fabricius, the first modern investigator to measure the tunnel, gave us an accurate description in 1884. Kastenbein in 1960, having used up-to-date surveying methods, made only minor corrections to Fabricius' measurements and added the important data of elevation which Fabricius had been unable to obtain.[6] The apparently unique and most intriguing feature of the tunnel—the fact that it was constructed from both sides according to an accurately calculated and precisely executed plan—had already been noted by Fabricius.

Tunnels and underground water-supply systems were nothing new to the sixth century B.C. A four-mile underground drainage channel from Lake Copais in Greece to the sea dates back to Mycenaean times. In Jerusalem, the spring of Siloam was diverted into the city underground when it was threatened by Senacherib, the Assyrian, in the eighth century B.C. That tunnel too was built from both sides to a meeting, as is attested by the oldest known Hebrew inscription, but it was a crude trial-and-error affair with many false starts in wrong directions and took 1,500 feet to cover a distance of 1,000 feet. An underground conduit system was built in Megara under the tyrant Theagenes late in the seventh century. It has been conjectured that Eupalinus, a native of Megara, was brought to Samos because of his experience with this type of construction in his hometown; we know nothing, however, about the tunnel in Megara. A tunnel in Athens, of similar construction and contemporary with the one on Samos, has not been explored far enough to learn much about the way it was planned, and there is no evidence to indicate that it was built from two sides.

The proof for the simultaneous construction from both ends of the Tunnel of Eupalinus is found in its greatest irregularity, which is the place of juncture near the center. Actually, the part of the tunnel dug from the south, after turning slightly to the right (east), comes to a dead end; the pick marks can still be seen on the blind head-wall that terminates it. Short of this wall the tunnel coming from the north, which has also turned east, breaks into the south tunnel through its western sidewall. In other

[5] Hero, *Dioptra*, Ch. 15. Throughout I will be referring to the edition by Hermann Schöne, *Herons Dioptra: Herons von Alexandria Vermessungslehre und Dioptra* (Leipzig: Teubner, 1902–1903).

[6] Kastenbein, "Eupalinos," p. 182.

words, a few feet before meeting, both tunnel halves turn somewhat to the east and thus meet almost at a right angle. In addition, at the meeting place, the floor of the north tunnel is about even with the ceiling of the south tunnel. Besides this difference in elevation, according to Kastenbein's calculations the two tunnels would have met head-on in a straight line had they not turned east just short of their meeting.

Kastenbein offers two possible explanations for this irregularity. In mining it is a common occurrence that sounds underground are distorted by the configuration of the rock formation so that they seem to come from a different location than their actual place of origin. Thus, the diggers might have heard each other, but could have been deceived into believing that the sounds were coming from the east. The second possibility is that because of an error in the builder's calculations it appeared that the tunnels should have met already, and consequently the two groups started looking for each other. A third possibility occurs to me: realizing that with the slightest directional error the two tunnels would bypass each other, the builder made the two work parties turn obliquely to the same side; then they would *have to meet* provided they were in the same plane. In any event, the error, intentional or not, is very minor: essentially, through almost their entire length, the two halves of the tunnel are perfectly in line, attesting to the accuracy of the builder's calculations. It should also be mentioned that according to Kastenbein's survey the elevation of the north entrance is 6 feet higher than that of the south exit.

The water pipes were laid in a channel along the east wall of the tunnel. Where the water channel enters the tunnel close to the north entrance, it was dug as a trench about 4 feet deep, but as its depth increases progressively to 25 feet at the south exit, it becomes a second tunnel driven through bedrock under the first, connected with the main tunnel by shafts at regular intervals and in places where its roof has collapsed. Fabricius was the first to raise the question of the need for this second tunnel and suggested that it had become necessary because an error in planning had provided insufficient slope for good flow within the tunnel. Fabricius was quite reluctant to impute such an error to an engineer who had been skillful enough to accomplish the successful junction under the mountain. Accordingly, he offers several explanations for the error: (a) construction of the underground reservoir might have been an afterthought when work on the tunnel had already started and could have lowered the source of the flow; (b) the wish to provide the highest possible terminal above the city, to facilitate the distribution of the water to all parts of town, might have induced the error. Fabricius also noticed that most of the covered water channels were about 6 feet high, just sufficient to allow passage of a man, but that at the south exit the channel was 9 feet deep. He considered this an indication that the channel was deepened later to improve the flow and also further evidence that the channel was dug afterward by trial and error, to correct the original underestimate of the degree of slope required.

Kastenbein, as well as Goodfield and Toulmin, quote and accept Fabricius' explanation. In doing so, however, they ignore the fact pointed out by Fabricius himself in a later article: that similar underground conduit systems in Syracuse, Acragas, and Athens show the same double-tunnel construction.[7] Doerpfeld, in 1894, called

[7] E. Fabricius, "Eupalinos," in Pauly-Wissova, *Realencyclopädie der Altertumswissenschaften*, Vol. VI (Stuttgart: J. B. Metzler, 1894–1963), Col. 1159.

attention to the similarity of the water system for the Enneacrounos in Athens, dating from the same period, to the one on Samos.[8] There, too, two tunnels were found, one above the other with the lower one containing the identical type of pipes as found in the Tunnel of Eupalinus; both tunnels were connected by vertical shafts. Thus, it becomes difficult to ascribe this building method to an error, and we rather must assume that it was intentional and customary. Indeed, Carl Curtius conjectured that ventilation was the reason for this type of construction. All Greek and Roman aqueduct systems have airshafts at regular intervals, This is true even as early as the Mycenaean system for Lake Copais, which had at least twenty airshafts to the surface.[9] It also should be noted that the covered trench-type channels to and from the Tunnel of Eupalinus were provided with regularly placed access shafts. Even Roman above-ground aqueducts had such shafts, and Vitruvius prescribes an opening every 120 feet.[10] Accordingly, it is Curtius' opinion that in places where it was impossible to reach the surface with airshafts (such as under a mountain) the upper tunnel was built to assure ventilation.

Curtius may be right and the need for ventilation may have played a part, but the primary reason for the shafts was certainly more basic. The original way to build an underground passage was to sink shafts and then connect them. In Asia Minor and Iran this is the method by which tunnels, the so-called *qanats*, are driven into the mountains to tap the groundwater trapped in the rock strata for irrigation purposes. It is a technique that has been continuously in use, possibly since the second millennium B.C.[11] The Mycenaean drainage tunnels for Lake Copais similarly connected shafts that were needed not only to bring the excavated material to the surface, but also subsequently for access to keep the passages free from obstructions. Where pipes or conduits were used, they had to be periodically maintained and replaced. Therefore, shafts probably came before the need for ventilation was recognized through experience in mining. Thus, it may well be that in the sixth century, when Pisistratus and Polycrates had their tunnels built, ventilation was considered, but the primary reason for the double construction was probably the difficulty of holding a constant slope of less than 1% in the depth of a mountain with primitive instruments. (Vitruvius[12] prescribes a minimum slope of 0.5%, but even in his time—the late first century B.C.—contemporary aqueducts show inexplicable error margins in slopes varying from 0.001% to 24.7% not caused by terrain factors.[13]) Various types of levels, however, were an effective aid in maintaining the horizontal. It was necessary to get through the mountain first in level tunneling; then, from this base, one could dig a sloping trench or second tunnel with the customary shafts by trial and error or by increasing the depth by a certain increment over each unit of distance.

Fabricius had no means of measuring elevation and assumed that the main tunnel had a slope. When Kastenbein's first professional triangulations showed that the north entrance is 6 feet higher in elevation than the south exit, Goodfield and Toulmin, who

[8] Wilhelm Doerpfeld, "Die Ausgrabungen an der Enneakrunos," *Athenische Mitteilungen*, 1894, *19*:144–146.

[9] As quoted by Augustus C. Merriam, "A Greek Tunnel of the 6th Century B.C.," *The School of Mines Quarterly* (New York), 1885, *4*:272.

[10] Vitruvius, *On Architecture* 8. 6. 7.

[11] R. J. Forbes, *Studies in Ancient Technology*, Vol. II (Leiden: Brill, 1965), p. 11.

[12] Vitruvius, *On Architecture* 8. 6. 1.

[13] Abbott Payson Usher, *A History of Mechanical Inventions* (Harvard: Harvard Univ. Press, 1966), p. 148.

made no measurements of their own, assumed that a slope must have been planned. But this 6-foot difference in elevation at the exits is the same difference as at the point of juncture at the center, where the floor of the north tunnel is level with the 6-foot-high ceiling of the south tunnel. Consequently, it is quite clear that the two tunnel halves are almost horizontal (actually they sag very slightly toward the center) and were meant to be horizontal.

III

The only clue from antiquity about how the tunnel might have been designed comes from Hero of Alexandria, in the first century A.D. In his *Dioptra*, Chapter 15, Hero poses the problem of digging through a mountain in a straight line when the mouths of the tunnel on the mountain are given. Later he indicates that the digging can be done from either side or from both. His last sentence in the chapter, however, indicates that he had construction from both ends especially in mind: if the digging is done in this manner, the workers will meet.[14] He solves the problem by means of the instrument which gives the book its name—the dioptra, which is a surveyor's instrument combining a theodolite with a water-level.[15] The instrument is highly sophisticated, probably invented by Hero himself, but seems never to have been used in antiquity.[16] Hero's proposed solution consists essentially in a series of right-angle offset horizontal sightings and measurements around the mountain. The angle of attack from both sides is determined by laying out on the ground, at the tunnel endpoints, two triangles similar to each other and to an imaginary triangle formed within the mountain by the actual line of the tunnel and an arbitrarily assumed base line.[17]

Hero mentions no specific tunnel, and no evidence exists in his writings that he had the Tunnel of Eupalinus in mind or that he even knew the tunnel. The temptation, nevertheless, is great to connect Hero's problem with our tunnel since Herodotus had made it the best known tunnel in the Greek world.[18] Relationships between Samos and Egypt had always been close, as we shall show later. Wilhelm Schmidt, in 1903, therefore suggested that the tunnel was designed as described by Hero,[19] and his suggestion was accepted by Bidez, van der Waerden, and others[20] and remained undisputed until recently. When Kastenbein and Goodfield and Toulmin, however, visited the tunnel, they raised serious doubts about this assumption.

Kastenbein made his survey in 1958 and published his findings in 1960. As a professional mining engineer, he made no pretenses of being a classicist or an historian of

[14] *Dioptra*, ed. Schöne, Vol. III (1903), pp. 238–241.

[15] This instrument has been described by A. G. Drachmann in "Dioptra" in Pauly-Wissova, *Realencyclopädie*, Suppl. Vol. VI, 1287–1290, and in his "Heron's Dioptra and Levelling-Instrument" in *A History of Technology*, Vol. III (Oxford: Clarendon Press, 1957), pp. 609–612.

[16] See A. G. Drachmann, *The Mechanical Technology of Greek and Roman Antiquity* (Copenhagen: Munksgaard, 1963), p. 198, and A. G. Drachmann, "A Detail of Heron's Dioptra," *Centaurus*, 1969, *13*: 243.

[17] B. L. van der Waerden, *Science Awakening*, trans. Arnold Dresden (Oxford: Oxford Univ. Press, 1961), pp. 103–104; also quoted in Goodfield and Toulmin, "How Was the Tunnel Aligned?" p. 49.

[18] *Cf.* Aristotle, *Politics* 5. 11, referring to the building activity of Polycrates on Samos.

[19] Wilhelm Schmidt, "Nivellierinstrument und Tunnelbau im Altertume," *Bibliotheca Mathematica*, 1903, Ser. *3–4*: 7–11.

[20] J. Bidez, *Eos, Platon et l'Orient* (Brussels: M. Hayez, 1945), p. 12; van der Waerden, *Science Awakening*, p. 103; Giorgio de Santillana, *The Origins of Scientific Thought, from Anaximander to Proclus, 600 B.C. to 300 A.D.* (Chicago: Univ. Chicago Press, 1961), p. 25.

science. He made a highly competent survey and judged from an engineering point of view how the layout of the tunnel could have been accomplished in the easiest fashion. If he was familiar with Hero's problem 15, he gave no indication in his first article. Kastenbein came to the conclusion that the easiest method of establishing the direction of the tunnel would have been to plant a row of poles over the mountain. The elevation of the endpoints could have been found by measuring up from a base line established by surveying around the western end of Mt. Kastro. This would have involved sighting along the south slope of the mountain toward the west at the edge of the coastal plain, then north through the streambed which separates Mt. Kastro from Mt. Kataruga—quite a tricky procedure, as Kastenbein says. In a second article, in 1966, however, Kastenbein acknowledges the feasibility of Hero's method and states that a decision as to which method was actually used is impossible.[21]

June Goodfield, writing in 1964, flatly denies that the tunnel could have been planned in the manner described by Hero.[22] Her conclusions are based on the following considerations:

- Hero's construction would not have solved the problem of slope.
- The roughness of the terrain would have made surveying around the west end of the mountain impossible.
- The site of the tunnel, far toward the western end of the city rather than above the city center, indicates that the location must have been chosen for a special reason. The reason for the choice, she believes, was the fact that this was the only place on the mountain where it was easy to run a straight line of poles over the ridge between the endpoints of the tunnel (as described by Kastenbein). To determine elevation one could sight horizontally from the foot of one pole to the next pole and measure the height intercepted. Thus the problem of distance and elevation would become simply one of addition and subtraction, without any need for more sophisticated mathematics:

> It has been suggested that his theory of similar triangles was applied to the building of the tunnel. The only evidence for this comes from Hero of Alexandria, who lived 600 years later. Hero was noted for thinking up charming theoretical solutions for difficult practical problems, and he gives as a theoretical exercise a method for aligning a tunnel.[23]

A year later in the *Isis* article written jointly by June Goodfield and her husband, Stephen Toulmin, they repeat essentially the same arguments, taking specific issue with van der Waerden's presentation of the view that the tunnel might have been built by use of the principles described by Hero, and also with the implication that "Pythagorean" mathematics might have played a part.[24]

Most recent scholars consider Hero not an elegant theorist but a practical engineer,[25] and his works handbooks for the working engineer and architect.[26] This is why Hero's

[21] W. Kastenbein, "Markscheiderische Messungen im Dienste der Archäologischen Forschung," *Mitteilungen aus dem Markscheidewesen*, 1966, *73*:26–36.

[22] June Goodfield, "The Tunnel of Eupalinus," *Scientific American*, 1964, *210* (No. 6):104-112.

[23] *Ibid.*, p. 112.

[24] Goodfield and Toulmin, "How Was the Tunnel Aligned?" p. 52.

[25] E.g., Edmond R. Kiely, *Surveying Instruments, Their History* (New York: Columbia Univ. Teachers College, Bureau of Publications, 1947), p. 19.

[26] Thomas Heath, *A History of Greek Mathematics*, Vol. II (Oxford: Clarendon Press, 1921), p. 307.

writings enjoyed popularity and influence into Roman times and the Middle Ages while the abstract mathematics of Archimedes was all but forgotten.[27] Giorgio de Santillana sees Hero expressing contempt for pure theory,[28] and Otto Neugebauer considers Hero a typical practitioner of the Egyptian-Mesopotamian mainstream of cooking-recipe mathematics which continued into the Arabic and Indian tradition, only superficially influenced by the axiomatic mathematics of the Hellenistic school.[29] It is for this reason that the objection of the six-hundred-year interval between the construction of the tunnel and Hero's writings loses much of its validity. The surveying methods that Hero described were the same that had been in use for many centuries.[30] In his *Dioptra* he only shows that all the ordinary problems, for which the *groma* and the leveling instruments with line and plummet were used, could be better solved by application of his instrument.[31] The *groma*, of Egyptian origin, and the *chorobates* both seem to predate the construction of the tunnel.[32] Thus it would appear that Hero is very much part of an engineering tradition that had not changed much in the intervening time.[33]

The view that the site was chosen because of its relative suitability for aligning a row of poles over the mountain is not fully convincing. The purpose of the tunnel was to bring a safe water supply within the city walls. So long as the tunnel exit was within the fortification, this purpose was achieved. From the tunnel's mouth a conduit had to be laid along the slope in an east-westerly direction, paralleling the length of the city, so that branch lines could be run downhill to feed all parts of the city. It would not have made any difference whether the tunnel ended above the city center and conduits had to be laid from there, both to the east and to the west, or if the tunnel came out in the west and a single longer conduit had to be run from there toward the eastern end. Essentially, the tunnel follows the straight and shortest line from the spring in Agiades to the closest point within the city walls. The only notable exception is the detour around the streambed below the north entrance. A move of the tunnel exit to the east would have lengthened the tunnel. The obvious objective was to keep the length of the tunnel to a minimum. The labor of digging a trench-type channel outside of the mountain was insignificant in comparison with tunnelling through solid rock. Consequently it would seem that the only determining factor in site selection was the required length of the tunnel.

Although the slope is quite steep and rocky, there is no doubt that the method of measuring *over* the mountain, proposed by Kastenbein and Goodfield and Toulmin, is quite possible. Is the other method of measuring *around* the mountain, as suggested by Hero, also feasible? In his second article Kastenbein did not rule out the possibility, and he refrained from making a judgment which would favor one method over the

[27] Otto Neugebauer, *The Exact Sciences in Antiquity* (Providence, R. I.: Brown Univ. Press, 1957), p. 146.

[28] De Santillana, *Origins of Scientific Thought*, p. 276. *Cf. Herons Belopoiica*, ed. H. Diels and E. Schramm (Abhandlungen der preussischen Akademie der Wissenschaften) (Berlin, 1918), Ch. 1.

[29] Neugebauer, *Exact Sciences*, p. 80.

[30] *Ibid.*

[31] Kiely, *Surveying Instruments*, p. 28.

[32] *Ibid.*, p. 14. The *groma* consists of a rectangular cross suspended horizontally at its fulcrum, with plumblines hanging down from the four endpoints, and serves to lay out directions at right angles to each other in the field. The *chorobates* is a levelling instrument shaped like a four-legged wooden bench; it may be levelled either by aid of plumblines suspended from it or by observance of the level of water in an appropriate depression in its upper surface.

[33] Usher, *Mechanical Inventions*, pp. 98–99.

other.[34] I believe the difficulties of the terrain on the west slope have been overstated. There is a considerable difference between difficult and impossible, and it is hard to tell where the line should be drawn. I was able to walk on a fairly horizontal traverse around the western end of Mt. Kastro in less than a hour. There are no overhanging rocks on the west slope, and whatever rocky outcroppings there are offered no serious obstacles to anyone willing to scramble over them. I also think it is a mistake to assume that the terrain has always been as we find it today. In 2,500 years erosion has taken its toll. In antiquity, before the hillsides were denuded of vegetation, they may have been covered with a thick layer of topsoil. A portion of the ancient bay has been filled by silt, and the modern town of Pythagorion is built on alluvial land that used to be part of the ancient harbor.[35] All this material can have come only from the mountain above the city whose slopes once stretched to the sea. Thus, we have no assurance that the terrain features in antiquity were the same we see today, but even if they were similar, they would not have offered any insurmountable obstacles. For instance, in plotting the similar triangles at the tunnel exits, it would have been possible to erect poles or rough wooden scaffolds to compensate for the differences in elevation and thus to obtain level measurements without resorting to "hundreds of vertical measurements."[36]

I also believe that much confusion has been created by drawings that have been reconstructed by modern editors to illustrate Hero's problem 15. Figure 1 shows Fabricius' map, whereas Figure 2 is a drawing by van der Waerden for his English version of *Science Awakening*, which was subsequently used by Santillana and then by Goodfield and Toulmin in their *Isis* article.[37] A comparison of Figures 1 and 2 shows instantly the influence of Fabricius' map. The outline of the mountain and the angle of the tunnel are the same. The important feature to note is that van der Waerden's drawing shows sightings around the *west* side of the mountain. (The author has apparently not seen the drawing in the codex.) Figure 3 is taken from Hermann Schöne's *Herons Dioptra*, which has been the standard edition ever since its publication in 1902. From this drawing the following is evident: Schöne either was not familiar with the tunnel on Samos or, more likely, he chose not to refer Hero's problem to any specific tunnel (he never mentions the Tunnel of Eupalinus). We know, however, that Schöne was familiar with the sketch in the codex (Fig. 4), and this is clearly reflected in his own drawing, which shows the sightings around the *eastern* end of the mountain. We also can see that the similar right triangles are constructed as they are in the codex, while van der Waerden's are reversed. Figure 4 is a reproduction of the actual drawing in the Mynas Codex (Paris, Bibliothèque Nationale, Supl. Grec No. 607), dating from the eleventh or twelfth century.[38] This is the primary manuscript from which all other existing ones are derived.[39] The drawing shows measurements around the *east* side of the mountain. Now, if one concedes any significance to

[34] Kastenbein, "Markscheiderische Messungen," *loc. cit.*

[35] Ulf Jantzen, "Samos 1966," *Archäologischer Anzeiger*, 1967, *82*:278.

[36] Goodfield and Toulmin, "How Was the Tunnel Aligned?" p. 52.

[37] Van der Waerden, *Science Awakening*, p. 103; de Santillana, *Origins of Scientific Thought*, p. 25; Goodfield and Toulmin, "How Was the Tunnel Aligned?" p. 49.

[38] Drachmann, *Mechanical Technology of Antiquity*, p. 163.

[39] Drachmann, "A Detail," p. 241.

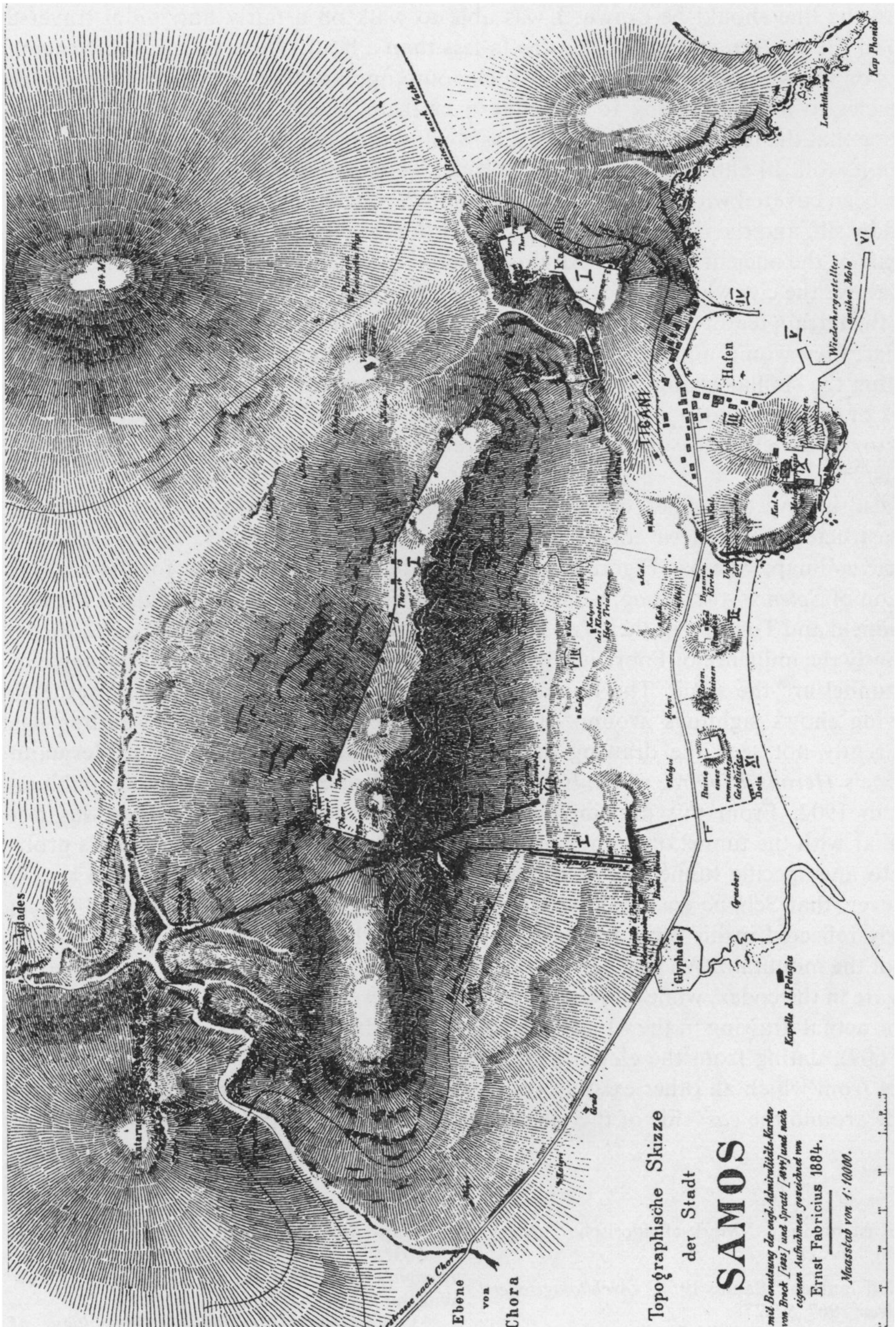

FIGURE 1. *Fabricius' topographical map showing Mt. Kastro and the location of the tunnel. (By permission of the German Archaeological Institute in Athens.)*

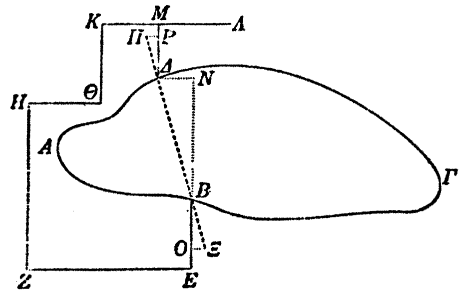

FIGURE 2. *Van der Waerden's drawing of the layout of the tunnel (from* Science Awakening, *Oxford: Oxford Univ. Press, 1961, p. 103).*

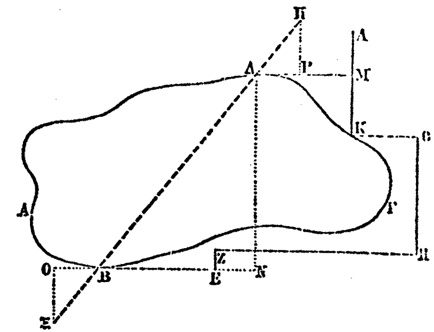

FIGURE 3. *Schone's drawing of the layout of the tunnel (from* Herons Dioptra, *Leipzig: Teubner, 1903, p. 238).*

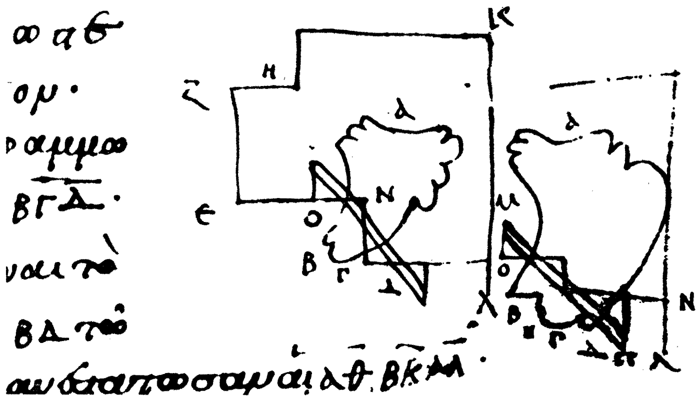

FIGURE 4

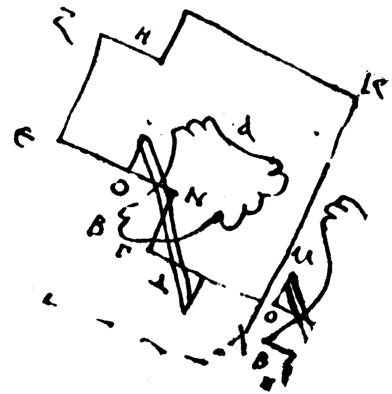

FIGURE 4a

FIGURE 4. *Photographic reproduction of the drawing in the Mynas Codex,* Dioptra, *Ch. 15; 4a is the drawing oriented with Fabricius' map. (By permission of the Bibliotheque Nationale, Paris.)*

the drawing in the codex, the argument—which is based on the difficulty of the terrain on the west slope of the mountain—collapses. The eastern circuit is somewhat longer but, I believe, a much easier route. A wide plain stretches from Agiades toward the east, where a relatively gentle saddle leads toward the south side of Mt. Kastro. From there, I believe, the traverse west to the exit of the tunnel is quite straight. I have to admit, however, that at the time of my visit I had not seen the codex drawing and was only familiar with van der Waerden's sketch. Consequently, like everyone else, I never investigated the eastern circuit around the mountain. An experimental survey over this route would be definitely required before any conclusions are drawn.

This brings us to the crucial question: can any weight as evidence be given to the manuscript drawing for or against the conjecture that Hero's problem refers to the Tunnel of Eupalinus? First we must ask the question whether there is any resemblance between the drawing and the actual situation on Samos. I believe that, allowing for a certain amount of distortion, sufficient similarities exist when we compare the codex drawing with Fabricius' map to conclude that the author of the sketch, or the author of the source from which this drawing was copied, appears to have been aware of the topography and the layout of the tunnel on Samos. The following resemblances seem too numerous to be ascribed to coincidence:

1. The direction of the tunnel in relation to the mountain is quite correct.
2. The location of the tunnel near the west end of the mountain is fairly correct.
3. The three fingerlike ridges jutting from the mountain toward the northwest can be clearly recognized on both the drawing and the map.
4. To a lesser degree this holds true for the cliffs at the southwest and east ends of the mountain.
5. The angles of the sightings around the east side of the mountain would be approximately correct.

The foreshortening of the east-west dimension and the exaggerated bulge of the mountain toward the north, as well as the shifting of the tunnel entrance to the southwest of the three finger ridges, would have to be ascribed to the distortion that is to be expected in repeated copyings by scribes, unfamiliar with the true situation and most likely unable to understand the subject matter. Some of the distortions, as A. G. Drachmann has pointed out to me, are readily explainable by the way the drawing was made. It seems the draftsman drew the straight lines first and then tried to fit the outline of the mountain onto them. But since he missed the important point of intersection between the tunnel line and the outline of the mountain at Delta, he tried a second time (in the drawing at the right), but failed this time to have his two similar right triangles originate at the Delta intersection. Because the angles are approximately 45°, Dr. Drachmann believes that the original from which the freehand sketch was copied must have been drawn with the help of a triangle: he considers that an indication that the drawing might have been a purely theoretical illustration. It seems to me, however, that the identifiable resemblances are too numerous to be all ascribed to coincidence, and it might have been the very use of a 45° triangle by one of the scribes that contributed to the distortion. I admit that all this is very much a matter of opinion, but so are all previous speculations concerning the tunnel, and the codex drawing may come closest to offering a tangible clue.

We may then sum up our findings so far:

(1) The identical difference of 6 feet between the elevation of the tunnel halves at

their endpoints as well as at the junction point indicates that the main tunnel was planned to be level. Because previous observers interpreted this difference in altitude as an attempt to provide a slope, they were prone to reject Hero's solution because it failed to provide an answer to the problem of slope.

(2) The second (lower) tunnel was dug to provide the slope. This double construction was intentional and customary (it is also found in other tunnels of the same period). The reason for the construction is probably the fact that no adequate instrument existed to hold a slope of less than 1 %, while tools such as the *chorobates* were effective in holding the horizontal. Ventilation may have been a factor; easy access certainly was.

(3) The tunnel could have been laid out by measuring over the mountain or around the east as well as the west slope.

(4) The argument that site selection offers a clue to the method of alignment is not convincing. If one accepts the drawing in the Mynas Codex as evidence that Hero had information about the tunnel, it seems that the surveying was done around the east end.

IV

Lastly, I would like to discuss the possibility of a connection between the tunnel and Pythagorean mathematics. Pythagoras' role in this field is a much-debated question and in fact is one of the reasons for the interest in the tunnel. Many scholars have considered Pythagoras a legendary sage, no more than a glorified medicineman,[40] while others believe that the early Ionion thinkers such as Pythagoras and Thales might have played the initial part in the development of Greek mathematics with which they are credited by the ancient tradition.[41] The purpose of the speculations concerning the tunnel is not to ascribe to Pythagoras an active part in the calculations, but rather to ascertain the state of mathematics in his age and area in order to determine the likelihood of his having discovered some of the theorems which antiquity connected with his name. After all, there is nothing in the mathematics ascribed to Pythagoras that was not part of the empirical knowledge of the Mesopotamians for the previous eight hundred years and subsequently of the Egyptians.[42] The question remains unanswered whether scientific mathematics (i.e., the system whereby each successive theorem requires rigid proof on the basis of previously recognized axioms or proven theorems) burst forth fully developed at the dawn of the fourth century or if it was the endproduct of an evolution that had begun when Greek thinkers first became acquainted with the handed-down mathematical procedures of the Near East. If we review Samos' relationship with Egypt, we shall find that the island was a most likely spot for early penetration by Egyptian mathematical knowledge.

Samos had become an important maritime power during the sixth century B.C. and had conquered many islands and mainland cities.[43] The Samians developed an effi-

[40] E.g., Walter Burkert, *Weisheit und Wissenschaft. Studien zu Pythagoras, Philolaos und Platon* (Nuremberg: Verlag Hans Carl, 1962), pp. 443–445, 141–142, 409.

[41] W. K. C. Guthrie, *The Greek Philosophers from Thales to Aristotle* (New York: Philosophical Library, 1950), p. 88; G. S. Kirk and J. E. Raven, *The Presocratic Philosophers* (Cambridge: Cambridge Univ. Press, 1957), pp. 229–230; James A. Philip, *Pythagoras and Early Pythagoreanism* (Toronto: Univ. Toronto Press, 1966), pp. 174 f.

[42] Neugebauer, *The Exact Sciences*, p. 36.

[43] Herodotus, *Histories* 3. 39.

cient type of vessel for their far-flung trade, the so-called Samaena, "more capacious and paunchlike, so that it is a good deepsea traveler and swift sailor too."[44] The Samians remained an important seapower into the fourth century, despite their defeat by Pericles; Thucydides could put the words into their mouths that "they had come within an inch of defeating the Athenians."[45]

Ample archaeological evidence for intensive interchange with Egypt has been found on Samos. A glance at the early Samian works of "orientalizing" art in the museum of Vathi immediately drives home the Egyptian influence: statuary, bronzes, and pottery of Samian make reflect their Egyptian models; sphynx and griffin motifs abound. In addition, numerous imported Egyptian artifacts dating from all periods have been found, especially in the Hera sanctuary.[46] Herodotus, of course, tells us of the close friendship between Polycrates and the "Egyptian king Amasis."[47] Bearing more directly on the subject of mathematics, Hultsch has shown that the unit of measurement employed in building the Hera temple on Samos was the royal Egyptian cubit.[48] The temple (about two miles from the tunnel exit), the breakwater of the port of Samos, and the tunnel are the three structures that elicited Herodotus' admiration because of their colossal scale ("the greatest works of all the Greeks"), which alone might indicate the Egyptian influence.

Now we have previously pointed out that the procedures described by Hero are the ones that had been in use for hundreds of years in connection with traditional instruments such as the *groma* and the *chorobates*, or its predecessor the plumbline level.[49] All these instruments have been shown to be of Egyptian origin. (The dioptra seems to be Hero's own invention and was apparently never used, as mentioned earlier.[50]) Such instruments were a necessity for construction of the temple as well as the tunnel. In this connection it is interesting that Pliny the Elder tells us that Theodorus was co-architect of the Hera temple, and, in a different context, he mentions the same Theodorus as the inventor of the *norma* and the *libella.*[51] The *norma* was the tool (essentially a carpenter's square) commonly used by the Romans to lay out right angles in the field; the *libella* was a plumbline, probably connected with a leveling instrument. The story is significant because it points to a tradition that the basic survey tools were "invented" on Samos in the sixth century B.C. We know, of course, that they had been in use in Egypt for a long time.[52] Since Hultsch has shown the use of the Egyptian measurement system, it is not unreasonable to assume that the implements to apply the measurements found their way to Samos at the same time.

In any event, from the close commercial and political relations between Egypt and Samos, and from evidence of some technological interchange, it would appear that Samos (like Miletus) was one of the crossroads where the Greeks had early access to the accumulated store of practical mathematical knowledge that was to an extent common to Mesopotamia and Egypt. This consideration, I believe, will shed some light on the interrelationship of "Pythagorean mathematics," the construction of the

[44] Plutarch, *Pericles* 26. 3.

[45] Thucydides, *History of the Peloponnesian War* 8. 76. 4.

[46] Ulf Jantzen, "Archäologische Forschungen auf Samos," *Bild der Wissenschaft*, 1967, *1*:49.

[47] Herodotus, *Histories* 3. 39–41.

[48] Friedrich Hultsch, *Griechische und Römische Metrologie* (Berlin: Weidmann, 1882), p. 551.

[49] Kiely, *Surveying Instruments*, pp. 10, 14, 19; also Usher, *Mechanical Inventions*, pp. 98–99, 147.

[50] See Drachmann, "A Detail," p. 243; also Kiely, *Surveying Instruments*, pp. 24–25.

[51] Pliny, *Natural History* 34. 83 and 7. 198.

[52] Kiely, *Surveying Instruments*, p. 10.

tunnel, and Hero's problems, even if we come to the reluctant conclusion that none of our findings in investigation of the tunnel is sufficient evidence to decide any of the more specific questions.

There is no clear evidence whether the tunnel was planned by measuring over the mountain or around the mountain. Both methods are possible, and Hero offers procedures for both; whether he knew which one was used is a moot question. Goodfield and Toulmin argue that Hero was "almost certainly in the same position as we are today: guessing how the work was done from circumstantial evidence alone."[53] To this van der Waerden replied that the plans and calculations for the tunnel would have been put into writing and that it is entirely possible that the original plan or a copy might have found its way into the library of Alexandria.[54] Both opinions are pure conjecture, and, I believe, beside the point.

If we can draw any positive conclusion at all, it might be this: during the sixth century Samos became familiar with the Egyptian mathematical tradition and, possibly through the Ionian mainland cities, with the Babylonian. The procedures used in laying out the tunnel are an integral part of the Near Eastern mathematical tradition, and so are the procedures described by Hero. The so-called Pythagorean mathematics is an offshoot of that same tradition. Even if no tangible piece of evidence from Eupalinus ever reached Hero, he was more than guessing; he knew and described the practical methods of a craft that had changed little in the intervening period and that was to continue with little change for centuries after. These traditional methods had become the basis of scientific mathematics when the Greek thinkers began to ask the question "why?"; but the traditional way of doing things remained unaffected by the abstract speculations that went on side by side with them.[55] That both branches of mathematics—the practical and the theoretical—gained a foothold in Samos is evidenced by an elaborate water clock on the agora of Samos described in a commemorative inscription,[56] and by the work of Aristarchos of Samos, who 200 years later developed the model of a heliocentric universe which was essentially the same as Copernicus' 1,750 years later.

[53] Goodfield and Toulmin, "How Was the Tunnel Aligned?" p. 52.

[54] B. L. van der Waerden, "Eupalinos and His Tunnel," *Isis*, 1968, *59*: 82–83.

[55] Van der Waerden, *Science Awakening*, pp. 83–94. See also my article "The Fragments of Philolaos and Aristotle's Account of Pythagorean Theories in Metaphysics A," *Classica et Mediaevalia*, 1966, *25*:93–128, and Neugebauer, *The Exact Sciences*, p. 146.

[56] Renate Tölle, "Uhren auf Samos," *Opus Nobile. Festschrift für Ulf Janzten* (Wiesbaden: Franz Steiner, 1969), pp. 169–170.